Exam Programme
VWO Mathematics A

The exam programme recognizes the following domains:

Domain A	Mathematical skills
Domain B	Algebra and combinatorics
Domain C	Functions and graphs
Domain D	Change
Domain E	Probability and statistics

The exam topics per domain

Domain A: Mathematical skills

The candidate is able to think mathematically. This includes ordering and structuring data, translating a problem to an algebraic equivalent, problem solving and the ability to manipulate formulas.

Domain B: Algebra and combinatorics

Subdomain B1 Algebra

The candidate is able to perform calculations with numbers and variables. Furthermore, the candidate is able to work with brackets.

The candidate knows:

- the difference between absolute and relative data.

The candidate is able to:

1. perform calculations with (and without) variables in which certain algebraic rules are used, including those for exponents and roots;
2. perform calculations with ratios, percentages and fractions;
3. work with (and expand) brackets;
4. simplify algebraic expressions;
5. convert units.

Subdomain B2 Combinatorics

The candidate is able to structure and schematize counting problems and use the result in calculations.

The candidate knows:

- what a factorial is.

The candidate is able to:

1. draw a tree diagram;
2. calculate the number of permutations and combinations;
3. identify a given problem as a counting problem;
4. come up for a strategy for a counting problem and use this strategy to solve the problem.

Domain C: Functions and graphs

Subdomain C1 Standard functions

The candidate is able to recognize and work with linear functions, quadratic functions, power functions, sine functions, exponential functions and logarithmic functions represented by a graph, a table or a formula.

The candidate knows:

- the following **standard functions** including the names:
  - $f(x) = ax + b$ (linear function),
  - $f(x) = ax^2 + bx + c$ (quadratic function),
  - $f(x) = a \cdot x^n$, where n is a rational number (power function),
  - $f(x) = b \cdot g^x$ and $f(x) = b \cdot \mathrm{e}^x$ (exponential function),
  - $f(x) = \log_g(x)$ (logarithmic function) and $f(x) = \ln(x)$ (natural logarithm),
  - $f(x) = \sin(x)$ (sine function);
- the following characteristics of the abovementioned standard functions:
  - maximum and minimum,
  - where the function is increasing and decreasing;
- the following characteristics of the graphs of the standard functions:
  - points of intersection with the x-axis and y-axis,
  - maximum and minimum points,
  - asymptotic behaviour;
- the following concepts regarding exponential functions: base, exponent, initial value, growth factor, growth percentage, half-life and doubling time;
- the following concepts regarding sine functions: amplitude, equilibrium and period.

The candidate is able to:

1. recognize (and work with) the standard functions and their graphs;
2. use the different representations of a function (formula, table, graph, text) in a given problem situation.

Subdomain C2 Functions, graphs, equalities and inequalities
The candidate is able to determine and manipulate formulas and function rules.
Furthermore, the candidate is able to draw the graph of a function.
The candidate can solve equations and inequalities either by applying algebraic techniques or by using a graphing calculator. The candidate is able to interpret what the solution of an equation or inequality means in the given context.

The candidate is able to:
1. use substitution to calculate values in a formula;
2. rewrite formulas using algebra;
3. use the properties of exponents, roots and logarithms in calculations;
4. determine the formula of a standard function (see subdomain C1) that fits a given problem situation;
5. perform a transformation on the graph of a standard function, e.g. a translation or a multiplication with respect to the x- or y-axis, and determine the formula that corresponds to the resulting graph;
6. use and recognize relationships of the form $y = a \cdot x$ (directly proportional) and $y = \frac{a}{x}$ (inversely proportional);
7. set up an equation or inequality using a table, a formula, a graph or text;
8. solve equations and inequalities using the graphing calculator;
9. find values using linear interpolation and linear extrapolation;
10. determine the function rule (and draw the graph) when two functions are: added ($f(x) + g(x)$), subtracted ($f(x) - g(x)$), multiplied ($f(x) \cdot g(x)$), divided ($\frac{f(x)}{g(x)}$) or composed ($g(f(x))$);
11. use a logarithmic scale;
12. qualitatively reason with formulas that contain two or more variables.

Domain D: Change

Subdomain D1 Sequences
The candidate is able to recognize and describe the behaviour of a sequence.
Furthermore, the candidate is able to do calculations with sequences, in particular with arithmetic and geometric sequences.

The candidate knows:
- the notation for sequences: u_n, where n can start at 0 or 1.

The candidate is able to:
1. determine whether a sequence of numbers is an arithmetic or geometric sequence;
2. determine a direct or recursive formula for an arithmetic or geometric sequence;
3. calculate the sum of (part of) an arithmetic or geometric sequence;
4. interpret expressions with the Σ-symbol;
5. work with a recursive and a direct formula;
6. recognize and determine a recursive formula that fits a given problem situation.

Subdomain D2 Slopes

The candidate is able to relate the change of a graph (or function) to a difference quotient or the slope of the graph.

The candidate knows:

- the relationship between the slope of a graph and the corresponding tangent line.

The candidate is able to:

1. determine whether a graph is increasing or decreasing;
2. determine whether the growth rate is increasing or decreasing;
3. calculate the average rate of change of a graph on an interval and interpret what this number means in a given context;
4. calculate the slope of a graph in a point and interpret what this number means in a given context.

Subdomain D3 Derivatives

The candidate is able to calculate the derivative of linear functions, quadratic functions, power functions, exponential functions and logarithmic functions and use the rules for differentiation. Furthermore, the candidate is able use the derivative to describe the change of a function and to calculate the extreme values.

The candidate knows:

- the following notations for the derivative: $f'(x)$ and $\frac{\mathrm{d}y}{\mathrm{d}x}$

The candidate is able to:

1. calculate the derivative of the standard functions (as mentioned in subdomain C1) (with the exception of $f(x) = \sin(x)$);
2. use the sum rule, the product rule and the quotient rule;
3. use the chain rule for differentiating functions of the form $g\big(f(x)\big)$, where f and g are standard functions;
4. relate the value of the derivative to the slope of the graph in a given point;
5. use the derivative to calculate (and verify) the extreme values of a function;
6. solve an optimization problem using differentiation;
7. interpret the derivative in a given context and use it to qualitatively reason about the change of the function.

Domain E: Probability and Statistics

Subdomain E2 Representation of data
The candidate is able to interpret data from a table or diagram and evaluate these data on their merit.

Subdomain E3 Quantification of data
The candidate is able to summarize data using measures of central tendency (mean, median and mode) and measures of variability (range and standard deviation) and interpret these numbers within the given context. The candidate is able to calculate these numbers using the graphing calculator.

Subdomain E4 Probability
The candidate is able to determine probabilities using diagrams, by using combinatorics or by using the rules for probability (the sum rule and the product rule). The candidate is able to distinguish between sampling with replacement and sampling without replacement.

Subdomain E5 Probability distributions
The candidate knows what a random variable is and is able to determine the probability distribution, the expected value and the standard deviation of a random variable.
The candidate can use the graphing calculator to perform calculations for binomially and normally distributed random variables. The candidate knows the rules for random variables, e.g. the $\sqrt{n}$-law.

Subdomain E6 Inferential statistics
Within a context, the candidate is able to perform a statistical test.

The candidate is able to:
1. determine whether the statistical test concerns a probability / proportion (binomial test) or a population mean (test for a population mean);
2. formulate a null hypothesis and an alternative hypothesis;
3. determine whether the testing procedure is one-sided or two-sided;
4. use the result of the experiment or sample to calculate the p-value;
5. interpret the p-value and draw a conclusion within the given context.

Exam VWO Math A

Formula sheet

Differentiation

rule	function	derivative
sum rule	$s(x) = f(x) + g(x)$	$s'(x) = f'(x) + g'(x)$
product rule	$p(x) = f(x) \cdot g(x)$	$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
quotient rule	$q(x) = \dfrac{f(x)}{g(x)}$	$q'(x) = \dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\left(g(x)\right)^2}$
chain rule	$k(x) = f\left(u(x)\right)$	$k'(x) = f'\left(u(x)\right) \cdot u'(x)$ or $\dfrac{\mathrm{d}k}{\mathrm{d}x} = \dfrac{\mathrm{d}f}{\mathrm{d}u} \cdot \dfrac{\mathrm{d}u}{\mathrm{d}x}$

Rules for logarithms

rule	condition
$\log_g(a) + \log_g(b) = \log_g(ab)$	$g > 0, g \neq 1, a > 0, b > 0$
$\log_g(a) - \log_g(b) = \log_g\left(\frac{a}{b}\right)$	$g > 0, g \neq 1, a > 0, b > 0$
$k \cdot \log_g(a) = \log_g(a^k)$	$g > 0, g \neq 1, a > 0$
$\log_g(a) = \dfrac{\log_p(a)}{\log_p(g)}$	$g > 0, g \neq 1, a > 0, p > 0, p \neq 1$

Sequences

The sum of an **arithmetic sequence** is given by:

$$S = \tfrac{1}{2} N (u_{\text{first}} + u_{\text{last}})$$

Here N is the number of terms.

The sum of a **geometric sequence** with common ratio r is given by:

$$S = \frac{u_{\text{last}+1} - u_{\text{first}}}{r - 1} \qquad \text{with } r \neq 1$$

Rules for random variables

For two random variables X and Y, we have:

$$\mathrm{E}(X+Y) = \mathrm{E}(X) + \mathrm{E}(Y)$$

For two independent random variables X and Y, we have:

$$\sigma(X+Y) = \sqrt{\left(\sigma(X)\right)^2 + \left(\sigma(Y)\right)^2}$$

If you have n independent random experiments, each with the same random variable X, then the following holds for the sum S and the mean $\bar{X}$:

$$\mathrm{E}(S) = n \cdot \mathrm{E}(X) \qquad \mathrm{E}(\bar{X}) = \mathrm{E}(X)$$

$$\sigma(S) = \sqrt{n} \cdot \sigma(X) \qquad \sigma(\bar{X}) = \frac{\sigma(X)}{\sqrt{n}}$$

Binomial distribution

For a binomially distributed random variable X, where n is the number of trials and p the probability of success, the probability of k successes is equal to:

$$\mathrm{P}(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$$

Furthermore: $\mathrm{E}(X) = n \cdot p$ and $\sigma(X) = \sqrt{n \cdot p \cdot (1-p)}$

Normal distribution

If X is normally distributed with mean μ and standard deviation σ, then:

$Z = \dfrac{X-\mu}{\sigma}$ follows a standard normal distribution with: $\mathrm{P}(X \leq g) = \mathrm{P}\left(Z \leq \dfrac{g-\mu}{\sigma}\right)$