Exam Programme
VWO Mathematics B

The exam programme recognizes the following domains:

Domain A	Mathematical skills
Domain B	Functions, graphs and equations
Domain C	Differential and integral calculus
Domain D	Trigonometric functions
Domain E	Coordinate geometry

The exam topics per domain

Domain A: Mathematical skills

The candidate is able to think mathematically. This includes ordering and structuring data, translating a problem to an algebraic equivalent, problem solving, the ability to manipulate formulas and the ability to provide a proof using logical reasoning.

Domain B: Functions, graphs and equations

Subdomain B1 Formulas and functions

The candidate is able to interpret and manipulate formulas. Furthermore, the candidate is able to draw a graph.

The candidate knows:

- the conditions under which a relationship between two variables is a function.

The candidate is able to:

1. rewrite a formula to an equivalent formula;
2. combine formulas into a new formula;
3. draw a graph given a relationship between two variables;
4. use a formula to make statements about the corresponding problem situation.

Subdomain B2 Standard functions

The candidate is able to recognize and draw graphs of the following **standard functions**: linear functions, quadratic functions, power functions with rational exponents, exponential functions, logarithmic functions, trigonometric functions and the absolute value function. In addition, the candidate knows and is able to use the characteristics of these functions.

The candidate knows:

- the graphs and the characteristics of linear functions $f(x) = ax + b$;
- the graphs and the characteristics of quadratic functions $f(x) = ax^2 + bx + c$;
- the graphs and the characteristics of power functions with rational exponents $f(x) = x^p$, in particular those of the square root function $f(x) = \sqrt{x}$;
- the graphs and the characteristics of exponential functions $f(x) = a^x$ and of logarithmic functions $f(x) = \log_a(x)$, also when the base equals e (so when $f(x) = \mathrm{e}^x$ and $f(x) = \ln(x)$);
- the graph and characteristics of the trigonometric functions $f(x) = \sin(x)$, $f(x) = \cos(x)$ and $f(x) = \tan(x)$, and the related concepts radian, period, amplitude and sinusoidal axis;
- the graph and characteristics of the absolute value function $f(x) = |x|$;
- the following properties of functions: domain, range, roots, extreme values, minimum, maximum, increasing and decreasing;
- the following properties of graphs: points of intersection with the x-axis and y-axis, minimum and maximum points, inflexion points, concave and convex, symmetry, asymptotic behaviour including horizontal, vertical and slant asymptotes.

The candidate is able to:

1. draw the graph of a standard function;
2. convert between the different ways of writing a quadratic function (e.g. $f(x) = ax^2 + bx + c$ and $f(x) = a(x-p)^2 + q$);
3. determine the function rule of a standard function when given a graph or a table;
4. use the characteristics of a standard function and its graph in solving problems;
5. describe an exponential function using the terminology 'initial value' and 'growth factor';
6. calculate the doubling time (or half-life) given exponential growth (or decay).

Subdomain B3 Functions and their graphs

The candidate is able to manipulate and combine function rules.
Furthermore, the candidate is able to draw the graph of a function and use the function rule to make qualitative statements about the function and its graph.

The candidate is able to:

1. translate a graph horizontally and vertically;
2. multiply a graph with respect to the x-axis or y-axis;
3. determine the formula corresponding to a graph obtained by a transformation (translation and/or a multiplication with respect to the x- or y-axis) of a given graph;
4. use the relationship between a transformation and the corresponding effect on the formula;
5. combine function rules by means of addition, subtraction, multiplication and/or division;
6. compose two functions $f(x)$ and $g(x)$ to obtain the composite function $g(f(x))$;
7. determine the characteristics of functions and their graphs;
8. determine a formula for a relationship between two variables described in a problem situation;
9. work with formulas and function rules that contain parameters.

Subdomain B4 Inverse functions

The candidate is able to determine, interpret and use the inverse function of a given standard function (see subdomain B2).

The candidate knows:

- the conditions under which a function has an inverse function.

The candidate is able to:

1. determine the inverse function of a power function, an exponential functions or a logarithmic function;
2. draw a graph of the inverse function when given the graph of the original function;
3. determine the inverse of a composite function;
4. use the characteristics of an inverse function and its graph in a given problem situation.

Subdomain B5 Equalities and inequalities

The candidate is able to solve and interpret equations and inequalities analytically.
This includes systems of two equations with two unknowns.

The candidate knows:

- what a system of equations is;
- the quadratic formula.

The candidate is able to:

1. solve an equation that can be reduced to a linear equation;
2. solve an equation that can be reduced to a quadratic equation;
3. solve an equation that can be reduced to $x^a = c$ or $|x| = c$;
4. solve an equation that can be reduced to $\log_a(x) = c$ or $a^x = c$;
5. solve an equation of the form $f(x) = g(x)$, where f and g are functions mentioned in subdomain B2;
6. solve an inequality of the form $f(x) > g(x), f(x) \geq g(x)$, $f(x) < g(x)$ or $f(x) \leq g(x)$, where f and g are functions mentioned in subdomain B2;
7. solve a system of two linear equations with two unknowns;
8. set up an equation (or inequality) for a given problem, solve the equation (or inequality) and interpret the solution(s) in the context of the original problem.
9. solve an equation that contains a parameter and write the solution as a function of the parameter;
10. solve an inequality of the form $f(x) < c$, $f(x) \leq c$, $f(x) > c$ or $f(x) \geq c$, where f is a composite function (as mentioned in B3.6)

Subdomain B6 Asymptotes and limits

The candidate is able to determine the asymptotic behaviour of a function. The candidate is able to prove this behaviour using limits.

The candidate knows:

- what a limit is (in the context of the behaviour of a function);
- what a left limit and a right limit are;
- the notation associated with limits.

The candidate is able to:

1. determine the asymptotes of the graphs of standard functions (as mentioned in subdomain B2);
2. use limits to investigate whether the graph of a function has a horizontal, vertical and/or a slant asymptote;
3. investigate whether the graph of a function has removable discontinuities.

Domain C: Differential and integral calculus

Subdomain C1 Derivative functions
The candidate is able to use the first and second derivative of a standard function (as mentioned in subdomain B2) to investigate the behaviour of the function. Furthermore, the candidate is able to use the first and second derivative in applications.

The candidate knows:

- the notations $f'(x)$ and $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the first derivative;
- the notations $f''(x)$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ for the second derivative.

The candidate is able to:

1. interpret the derivative as the slope of the graph in a point;
2. use the derivative to determine an equation of a tangent line to the graph of f;
3. use the derivative to determine whether a function is increasing or decreasing;
4. use to derivative to determine (and verify) the extreme values of a function;
5. sketch the graph of the derivative given the graph of the function itself (and vice versa);
6. use the second derivative to determine whether a graph is concave or convex;
7. use the second derivative to calculate inflection points;
8. solve an optimization problem using the derivative or the graphing calculator.

Subdomain C2 Techniques for differentiation
The candidate is able to determine the first and second derivative of a standard function (as mentioned in subdomain B2) using the rules of differentiation and algebraic techniques.

The candidate knows:

- the derivative of the standard functions;
- that differentiation means ‘the process of calculating the derivative’.

The candidate is able to:

1. use the derivatives of the standard functions to determine other, more complex, derivatives;
2. use the number e and the natural logarithm to determine the derivative of exponential and logarithmic functions;
3. use the sum rule, product rule, quotient rule or the chain rule to determine derivatives (or a combination of those rules);

Subdomain C3 Integral calculus

The candidate is able to, in certain situations, set up an integral and calculate the integral analytically.

The candidate knows:

- what an integrand, antiderivative and definite integral are;
- the notation $\displaystyle\int_a^b f(x)\,\mathrm{d}x$;
- the fundamental theorem of calculus:
  $\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$, where F is an antiderivative of f.

The candidate is able to:

1. calculate a definite integral analytically when the integrand is of the form $c \cdot f(x) + d$ or $f(c \cdot x + d)$ and f is a linear function, a quadratic function, a power function, an exponential function or a sine or cosine function (also when the integrand is a sum of two or more of these functions);
2. approximate a definite integral using the graphing calculator;
3. check whether a given function F is an antiderivative of a function f;
4. set up a definite integral for calculating the area of a region (e.g. the area under a graph or the area between two graphs);
5. set up a definite integral for calculating the volume of a solid of revolution, where the solid of revolution is obtained by revolving a region around the x-axis or y-axis;
6. interpret the value of a definite integral;
7. interpret $F(x) = \int_a^x f(t)\,\mathrm{d}t$ as a function of x.

Domain D: Trigonometric functions

The candidate can construct formulas for periodic phenomena and draw the corresponding graphs. Furthermore, the candidate can solve trigonometric equations.

The candidate knows:

- the sine model in the forms $f(x) = a + b \cdot \sin\big(c(x-d)\big)$ and $f(x) = a + b \cdot \cos\big(x(c-d)\big)$ and that the graph of these functions is called a sinusoid;
- the values of $\sin(x)$, $\cos(x)$ and $\tan(x)$, where x is a multiple of $\frac{1}{6}\pi$ or $\frac{1}{4}\pi$;
- the following trigonometric identities:
    - $\sin^2(x) + \cos^2(x) = 1$
    - $\tan(x) = \frac{\sin(x)}{\cos(x)}$
    - $-\sin(x) = \sin(x-\pi)$ (and $-\sin(x) = \sin(-x)$)
    - $-\cos(x) = \cos(x-\pi)$ (and $-\cos(x) = \cos(\pi - x)$)
    - $\sin(x) = \cos\left(\frac{1}{2}\pi - x\right)$
    - $\cos(x) = \sin\left(\frac{1}{2}\pi - x\right)$

The candidate is able to:

1. convert degrees to radians (and vice versa);
2. draw a sinusoid;
3. determine the formula that corresponds to a given sinusoid;
4. rewrite formulas using the abovementioned trigonometric identities;
5. solve equations of the form $\sin(x) = c$, $\cos(x) = c$ and $\tan(x) = c$, using periodicity and symmetry when needed;
6. solve equations of the form $f(x) = c$, where $f(x)$ is a sine model, using periodicity and symmetry when needed;
7. solve equations of the form $\sin\big(f(x)\big) = \sin\big(g(x)\big)$, $\cos\big(f(x)\big) = \cos\big(g(x)\big)$ and $\tan\big(f(x)\big) = \tan\big(g(x)\big)$, where f and g are linear functions, using periodicity and symmetry when needed;
8. solve inequalities of the form:
    - $\sin\big(f(x)\big) < c$, $\sin\big(f(x)\big) \leq c$, $\sin\big(f(x)\big) \geq c$ and $\sin\big(f(x)\big) > c$ where f is a linear function
    - $\cos\big(f(x)\big) < c$, $\cos\big(f(x)\big) \leq c$, $\cos\big(f(x)\big) \geq c$ and $\cos\big(f(x)\big) > c$ where f is a linear function
    - $\tan\big(f(x)\big) < c$, $\tan\big(f(x)\big) \leq c$, $\tan\big(f(x)\big) \geq c$ and $\tan\big(f(x)\big) > c$ where f is a linear function;
9. set up a sine model for a given periodic phenomenon, draw the corresponding sinusoid and use this sine model in calculations;
10. set up a sine model for a harmonic oscillation and use the terminology ‘frequency’ and ‘period’;
11. use the sum- and difference formulas and the double angle formulas to rewrite formulas and to solve equations;
12. use the abovementioned trigonometric identities to solve equations;

Domain E: Coordinate geometry

Subdomain E1 Geometrical skills
The candidate is able to determine geometrical properties of objects and formulate proofs using geometrical and algebraic techniques.

The candidate knows:

- the notion of distance as the length of the shortest line segment between two geometric objects;
- the following geometric theorems:
  - In a right-angled triangle the midpoint of the hypotenuse is the centre of the circumscribed circle.
  - A triangle of which a side is the centre of the circumscribed circle, is a right-angled triangle.
  - A tangent line to a circle is perpendicular to the radius (the line segment from the centre to the point of tangency).
  - If one draws two tangent lines from a point (outside a circle) to that circle, then the distances from that point to the two points of tangency are equal.
  - The tangent line in the common point of tangency of two tangent circles is perpendicular to the line connecting the two centres of the circles.
  - For each point P on the perpendicular bisector of line segment AB, the distance from P to point A is equal to the distance from P to point B.
  - For each point P on the angle bisector of lines l and m, the distance from P to line l is equal to the distance from P to line m.

The candidate is able to:

1. use Pythagoras' theorem to calculate the distance between two points;
2. use similarity to calculate the length of line segments;
3. use the sine, cosine and tangent to calculate the size of angles and the length of sides in right-angled triangles;
4. use the sine rule and cosine rule to calculate the size of angles and the length of sides in triangles;
5. describe a (part of a) geometric object algebraically;
6. draw a picture given a description of a geometric problem;
7. use geometric techniques to investigate and prove properties of geometric objects;
8. use algebraic techniques to investigate and prove properties of geometric objects.

Subdomain E2 Algebraic methods in planar geometry

The candidate is able to determine properties (and the relative location) of points, lines, circles and other geometric figures using algebraic representations in a suitable coordinate system. The candidate is able to use these algebraic representations to solve problems.

The candidate knows:

- the equation of a line in the forms $y = mx + n$ and $ax + by = c$;
- what is meant by the 'slope of a line';
- what a perpendicular is;
- the property that two lines are perpendicular if the product of their slopes is -1 (and vice versa);
- what a system of equations is;
- the notion of a dependent and an inconsistent system of equations;
- the equation of a circle in the forms $(x - a)^2 + (y - b)^2 = r^2$ and $x^2 + y^2 + ax + by + c = 0$;
- the parametric representation of a line:
  $\begin{cases} x(t) = a \cdot t + c \\ y(t) = b \cdot t + d \end{cases}$
- the parametric representation of a circle:
  $\begin{cases} x(t) = p + r \cdot \cos(t) \\ y(t) = q + r \cdot \sin(t) \end{cases}$

The candidate is able to:

1. calculate the distance between points, lines and circles;
2. calculate the angle between two lines;
3. determine the equation or parametric representation of a line;
4. determine the equation or parametric representation of a circle;
5. determine the coordinates of the centre of a circle and the radius given the equation of the circle;
6. convert a parametric representation of a line or circle to an equation (and vice versa);
7. calculate the coordinates of the points of intersection of two lines, two circles or a line and a circle;
8. describe the relationship between the solvability of a system of two linear equations and the relative location of the corresponding straight lines in the xy-plane;
9. determine the number of points that a line and a circle have in common;
10. determine the equation of a tangent line to a circle given the slope;
11. determine the equation of a tangent line through a given point (on the circle or outside the circle) to a circle;
12. solve geometric problems using the abovementioned algebraic techniques.

Subdomain E3 Vectors and the dot product

The candidate is able to use vectors and dot products in calculations and for determining certain properties of geometric objects in the xy-plane.

The candidate knows:

- the concept of a vector as an object with a length and a direction, notation:
  $\vec{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix}$
- the concepts length and components of a vector;
- the concept angle of inclination;
- the dot product of two vectors:
  $\vec{a} \cdot \vec{b} = a_x \cdot b_x + a_y \cdot b_y = |\vec{a}| \cdot |\vec{b}| \cdot \cos(\theta)$, where θ is the angle between $\vec{a}$ and $\vec{b}$;
- the vector representation of a line:
  $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix} + t \cdot \begin{pmatrix} a \\ b \end{pmatrix}$
  where $\binom{c}{d}$ is the position vector and $\binom{a}{b}$ is the direction vector;
- the centroid of a number of points as the endpoint of the vector that is the weighted mean of the vectors corresponding to the points.

The candidate is able to:

1. determine the vector representation of a line;
2. perform calculations with vectors that are described by their length and direction or by their components;
3. resolve vectors into their x- and y-components;
4. multiply a vector by a number and add two vectors, both geometrically and algebraically;
5. use the dot product to calculate angles and distances;
6. convert between an equation, a parametric equation and a vector representation of a line;
7. perform calculations that involve the path of a moving point described by a time-dependent vector representation: $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$
8. calculate the velocity vector and the acceleration vector of a moving point;
9. calculate the linear speed and the linear acceleration of a moving point;
10. determine the equation of a tangent line to the path of a moving point;
11. use vectors to determine centroids.

Exam VWO Math B

Formula sheet

Trigonometry

$\sin(t+u) = \sin(t)\cos(u) + \cos(t)\sin(u)$

$\sin(t-u) = \sin(t)\cos(u) - \cos(t)\sin(u)$

$\cos(t+u) = \cos(t)\cos(u) - \sin(t)\sin(u)$

$\cos(t-u) = \cos(t)\cos(u) + \sin(t)\sin(u)$

$\sin(2t) = 2\sin(t)\cos(t)$

$\cos(2t) = \cos^2(t) - \sin^2(t) = 2\cos^2(t) - 1 = 1 - 2\sin^2(t)$