BOSWELL-BÈTA

James Boswell Exam
VWO Mathematics A – Practice exam 1

Date:	
Time:	3 hours
Number of questions:	6
Number of subquestions:	24
Number of supplements:	0
Total score:	72

- Write your name on **every sheet of paper** you hand in.
- For each question, show how you obtained your answer either by means of a calculation or, if you used a graphing calculator, an explanation. Otherwise, no points will be awarded to your answer.
- Make sure that your handwriting is legible and write in blue or black ink. No correction fluid of any kind is permitted. Use a pencil only to draw graphs and geometric figures.
- You may use the following:
  - Graphing calculator (without CAS);
  - Drawing utensils;
  - List of formulas;
  - Dictionary, subject to the approval of the invigilator.

Question 1: Ice skating

Pim and Rafa are ice skaters. They are training at the ice skating facility Thialf. They skate rounds of 400 meter. To keep one of them out of the wind during these rounds, they position themselves precisely one after another. During each round either Pim skates in front of Rafa, or Rafa skates in front of Pim. During a normal training they skate eight rounds, such that they both skate an equal number of times in front of the other person.

2p **a** Suppose Pim starts in front of Rafa in the first round. Calculate in how many ways they can do the normal training.

Just before the start of the match season, Pim and Rafa are allowed to train with the professionals Sven, Antoinette and Kjeld. Also here they ice skate behind one another, forming a 'train' of ice skaters.

3p **b** How many 'trains' are possible in which Sven and Kjeld are positioned precisely in front of each other?

After the training, Rafa and Pim have five ice skating competitions, where they have each other as the opponent. The probability that Rafa wins a single competition is equal to 0.6.

2p **c** Calculate the probability that Rafa wins more than two of these competitions. Round your answer to three decimals.

A week later the two friends compete over a tournament cup. They use the *best of five* rule. This means that the ice skater who is the first to win three competitions wins the tournament cup.

The number of competitions needed before the tournament is decided is called X. In the table below you see the probability distribution for X. Two probabilities are missing.

k	3	4	5
$P(X = k)$	…	…	0.3456

4p **d** Calculate the probabilities missing from the table and determine the expected value of X. Round the expected value to one decimal.

Professional ice skater Antoinette often skates a distance of 3000 meters. This distance consists of one part of 200 meters, followed by 7 complete rounds of 400 meters. During the training Antoinette completes the first part of 200 meters in 20.6 seconds. The first complete round after that is performed in 30.4 seconds. The time she needs to complete a consecutive round then increases with 0.7% each round.

The time (in seconds) needed to complete the first full round will be called u_1.

2p **e** Determine a direct formula for u_n, where u_n is the time (in seconds) she needs to complete the n^{th} full round.

3p **f** Calculate whether or not Antoinette completes the entire 3000 meter training within 4 minutes.

Question 2: Vocabulary

The number of words that an adult Dutch person knows varies between $45\ 000$ and $250\ 000$ words. Research has shown that a child knows $17\ 000$ words on average on his 12th birthday. The largest differences in vocabulary develop after the 12th birthday.

For children with a **big** vocabulary it turns out that the amount of words they know grows exponentially after their 12th birthday. The formula describing the number of words they know is given by:

$$W_B = 17\ 000 \cdot \mathrm{e}^{0.239t}$$

Here t stands for time in years, with $t = 0$ at the 12^{th} birthday.

3p **a** Calculate with which percentage the vocabulary grows each month, according to this formula. Round your answer to one decimal.

Humam knows a lot of words for his age. He just turned 14 years old and knows about $42\ 500$ words.

4p **b** How many months is Humam younger than the expected age at which he is supposed to know this number of words? Use the formula for children with a big vocabulary. Round your answer to the nearest whole number.

For children with a **small** vocabulary it turns out that the number of words they know increases linearly after their 12th birthday. On their 21st birthday they know $45\ 000$ words on average. For children with a small vocabulary the number of words can be described by a formula of the form $W_S = at + b$.

2p **c** Write down this formula. Clearly explain your answer. Again, let t be time in years with $t = 0$ corresponding to the 12^{th} birthday. Round a to the nearest whole number.

3p **d** Calculate the age at which the number of words known by a person with a big vocabulary is twice the number of words known by a person with a small vocabulary.

Question 3: Derivatives

4p **a** Let the function $g(x) = \frac{10 \ln(x)}{x}$ be given.
Line k is tangent to the graph of g at point $A(1,0)$.

Use the derivative of g to determine a formula for line k.

6p **b** Let the function $h(x) = \mathrm{e}^{-0.75x} \cdot x^3$ be given.

Use differentiation to calculate the x-coordinate of the maximum and minimum point(s) of the graph of h. Indicate for each x-coordinate whether the corresponding point is a maximum or a minimum point.

Question 4: Cucumbers

In 1988 the European Union determined norms for the properties of cucumbers that will be used for consumption. One of the norms states that cucumbers must at least have a length of 25 cm.

A certain cultivator grows cucumbers. The length of his cucumbers is normally distributed with a mean of 36.2 cm and a standard deviation of 5.7 cm.

2p **a** Which percentage of his cucumbers is in agreement with European standards? Round your answer to one decimal.

European law also determines that the cucumbers have to weigh at least 180 grams. The weight of the cucumbers produced by the cultivator is normally distributed with a mean of 216.5 gram. He has determined that 10% of his cucumbers does not agree with European law.

3p **b** Calculate the standard deviation of the weight of cultivators cucumbers. Round your answer to one decimal.

In 1988 it has also been decided how much the cucumbers may be bent. This decision has led to a lot of criticism. It has been therefore been decided in 2009 to abandon the decision. So since 2009 bent cucumbers may be sold again.

A supermarket of the city Wageningen investigated if it is possible to stimulate the selling of bent cucumbers. During the investigation bent cucumbers were posited next to straight cucumbers. A large sign was put above the counter saying that a bent cucumber tastes as good as a straight one and that buying bent cucumbers prevents waste of food. During 14 days the selling of bent cucumbers has been recorded.

Translation: Bent cucumbers often do not end up on our plate due to their appearance. "I am just as tasty as my straight brother". Prevent waste and choose bent cucumbers.

Before the experiment, so without the sign above the counter, 30% of the sold cucumbers were bent. The supermarket suspected that the placement of the sign above the counter would have a positive effect on the sales of bent cucumbers. The supermarket tested this hypothesis using a statistical test.

2p **c** Give the hypotheses used in this statistical test.

Of the 2958 cucumbers sold during the 14 days, 1053 cucumbers were bent.

4p **d** Perform the statistical test and give the conclusion the supermarket will draw. Use a significance level of 5%.

Question 5: Mass and metabolism

In this question we will look at the energy usage of mammals at rest. The energy usage will be expressed in Watt (W = Joule per second).

The energy usage of big animals is higher than that of small animals. A dog, weighing 11.5 kg on average, has an energy usage of 25.6 W. A horse, weighing 650 kg on average, has an energy usage of 530 W.

2p **a** Show that, based on these facts, the relationship between the energy usage and the weight of an mammal is **not** directly proportional.

The American veteranarian and researcher Max Kleiber investigated the energy usage for animals of various weight in 1932. Based on his results he formulated the following relationship:

$$E = 4.1 \cdot m^{a}$$

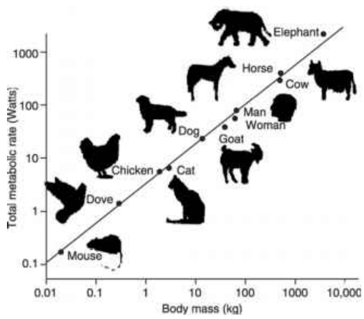
Here E is the energy usage in W and m is the weight of the mammal in kg. He determined the value of a to be approximately 0.75.

3p **b** Determine the value of a accurate to three decimals. Use the weight and energy usage of a dog at rest to determine this value.

3p **c** Calculate how many kilograms a mammal weighs if it uses 100 W of energy, according to this model. Round your answer to the nearest whole number.

Kleiber put his results on log-log paper. This means that he used a logarithmic scale on both axes. The points corresponding to his results lie approximately on a straight line. The relationship between E and m can therefore be written in the form:

$$\log(E) = a + b \cdot \log(m)$$



You can rewrite the formula $E = 4.1 \cdot m^{0.75}$ also into this form. The first step is:

$$\log(E) = \log(4.1 \cdot m^{0.75})$$

3p **d** Finish rewriting the formula into the form $\log(E) = a + b \cdot \log(m)$. Round a and b to two decimals if necessary.

In practice it is useful to determine the energy usage of an mammal per kilogram weight. This gives the so-called relative energy usage E_r (in W per kg).

In a formula:

$$E_r = \frac{E}{m}$$

2p **e** Show that $E_r(m) = 4.1 \cdot m^{-0.25}$. Clearly explain your answer.

This question continues on the next page

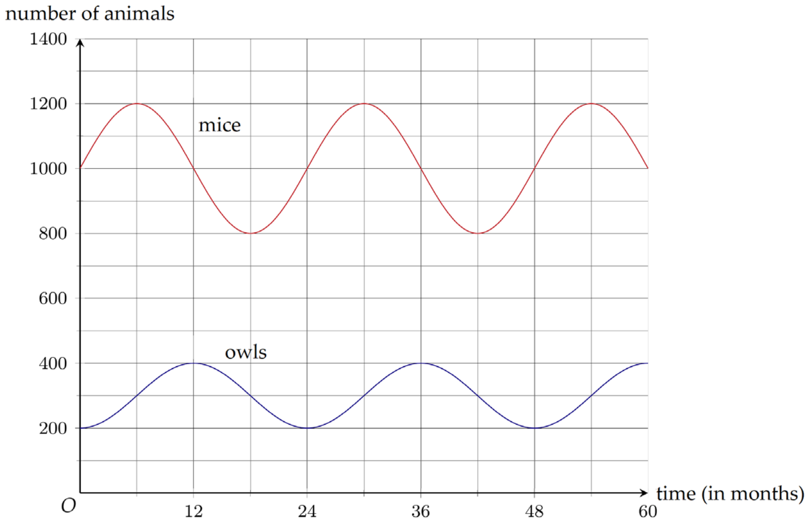
From the formula $E_r(m) = 4.1 \cdot m^{-0.25}$ you can deduce that heavy mammals use less energy per kilogram than light mammals. In other words: E_r is decreasing as a function of m.

3p **f** Use the derivative of E_r to show that the graph of E_r is decreasing for all values of m.

Question 6: Owls and mice

In a certain area the population of owls and mice varies according to a so-called *predator-prey cycle*. When there are many mice, owls will be drawn to occupy the area. After a while the increased number of owls will cause a decrease in the number of mice. The decrease in number of mice will then lead to a withdrawal of owls from the area which causes the number of mice to increase again. This process repeats itself over and over.

Both the population of owls and mice can be described by a sine function. In figure below you see the graphs of both populations.



The population of mice is described by the function $M(t) = 1000 + 200 \cdot \sin\left(\frac{1}{12}\pi t\right)$. Here $M(t)$ is the number of mice at time t. Time t is measured in months with $t = 0$ corresponding to 1 January 2015.

4p **a** Calculate in which months of the year 2015 there were over 1150 mice **during the entire month.**

The owl population can also be described by a sine function: $O(t) = a + b \cdot \sin\big(c(t - d)\big)$. Here t stands for time in months with $t = 0$ corresponding to 1 January 2015.

3p **b** Determine values for a, b, c and d. Clearly explain how you obtained your results.