

James Boswell Exam

VWO Mathematics A – Practice exam 2

Date:	
Time:	3 hours
Number of questions:	6
Number of subquestions:	24
Number of supplements:	1
Total score:	72

- Write your name **on every sheet** of paper you hand in.
- Use a separate sheet of paper for each question.
- For each question, show how you obtained your answer either by means of a calculation or, if you used a graphing calculator, an explanation. Otherwise, no points will be awarded to your answer.
- Make sure that your handwriting is legible and write in blue or black ink.
- No correction fluid of any kind is permitted. Use a pencil only to draw graphs and geometric figures.
- You may use the following:
 - Graphing calculator (without CAS);
 - Drawing utensils;
 - List of formulas;
 - Dictionary, subject to the approval of the invigilator.

Question 1: Eggs

An **egg code** is printed on each egg produced in Europe. An example of such a code is 2-NL-1234567.

The egg code consists of three parts:

- The first part is a number that indicates the production method of the eggs: 0 (organic chickens), 1 (free-range chickens), 2 (barn chickens) or 3 (caged chickens);
- The second part of the code indicates in which country the egg was produced. For example, NL (the Netherlands). In total there are 25 country codes;
- The third part consists of seven digits. The first five digits form a company code (a number ranging from 10 000 to 99 999). The last two digits refer to a particular chicken shed (ranging from 00 to 99).



- 3p **a** Calculate the number of possible different egg codes.

The Netherlands exports a lot of eggs to Germany. In 2018 the total export of eggs to Germany was worth 409 million euros, an increase of 13.6% compared to the previous year.

- 2p **b** Calculate by how many million euros exports to Germany increased between 2017 and 2018. Round your answer to the nearest million.

In Europe, the consumption of eggs has been increasing for quite some time. In 2013 Europeans ate an average of 198 eggs¹ per person. Since then, the egg consumption increased linearly to 216 eggs per person in 2019.

The average number of eggs consumed per person per year in Europe, can be described by a sequence N_t . Here t is the number of years with $t = 0$ in 2013.

- 3p **c** Determine both a direct and a recursive formula for the sequence N .

Assume that egg consumption in Europe will continue to increase in the same way after 2019.

- 3p **d** In which year will the average number of eggs eaten per person be more than 250? Explain your answer.

¹ This number includes the number of eggs used in pastries, ice cream, cakes and sauces.

Question 2: Cows

A farmer owns a farm with hundreds of cows. He suspects that a certain disease has broken out on his farm and decides to test his cows.



The test has two possible outcomes:

- *positive*: according to the test the cow has the disease;
- *negative*: according to the test the cow does not have the disease.

The test is not perfect though:

- If a cow has the disease, there is a 85% probability that the cow tests positive. This means that 15% of the sick cows get a negative test result;
- If the cow does not have the disease, there is a 98% probability that cow tests negative. This means that 2% of the healthy cows will get a positive test result.

Assume for questions **a**, **b** and **c** that 1 out of 4 cows on the farm has the disease.

- 2p **a** Calculate the probability that a randomly selected cow does not have the disease, but tests positive.

The probability that a randomly selected cow will test positive is equal to 0.2275.

- 3p **b** Show this by means of a calculation.

The farmer tests twenty cows. X is the number of cows that test positive.

- 4p **c** Calculate $P(2 < X \leq 8)$. Round your answer to three decimals.

The farmer knows from experience that his cows eat an average of 70.0 kg of food per day with a standard deviation of 12.5 kg. Assume that the amount of food a cow consumes per day is normally distributed.



The farmer suspects that sick cows on average eat less than healthy cows. He decides to investigate this by means of a statistical test. Of thirty cows, which have been diagnosed with certainty to be sick, a record is kept of how much they eat on a particular day.

- 2p **d** Formulate the hypotheses for this statistical test.

The record shows that the thirty cows have eaten an average of 62.3 kg.

- 5p **e** Perform the statistical test and formulate the conclusion the farmer will draw. Use a significance level of 5%.

Question 3: Derivatives

4p **a** Consider the function $f(x) = \frac{6}{\sqrt[3]{x}} - x \cdot \sqrt{x}$

Determine the derivative of f . Write your answer without negative or fractional exponents.

6p **b** Consider the function $g(x) = \frac{e^{2x-1}}{x}$

Use differentiation to find the coordinates of the maximum and/or minimum point(s) of the graph of g .

Question 4: Mushrooms

A mushroom farm grows several types of mushrooms, including portobellos.

The diameter of the portobellos is normally distributed with a mean of 11.2 cm and a standard deviation of 0.6 cm.



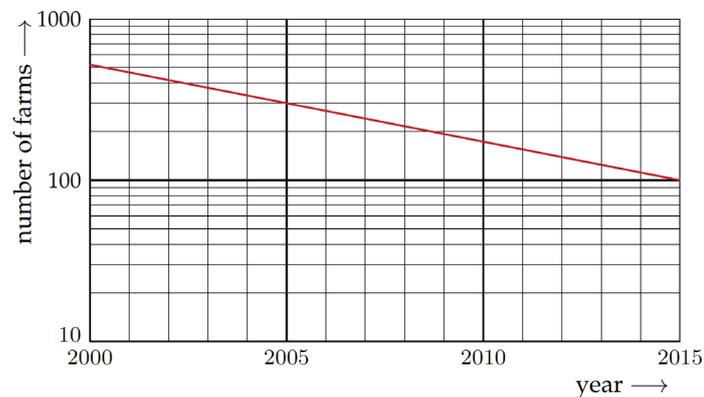
- 2p **a** Calculate how many cm the diameter of a portobello is at least if it belongs to the 10% with the largest diameter. Round your answer to the first decimal.

The mushroom farm also grows white button mushrooms. The weight of these mushrooms is normally distributed with a mean of 13.0 grams and a standard deviation of 1.9 grams.

The button mushrooms are randomly put in packages of 250 grams. The probability that the total weight of the mushrooms in such a package is less than 250 grams depends on the number of mushrooms in the package.

- 4p **b** Determine the minimum number of mushrooms in a package, so that the probability that the total weight of the mushrooms is less than 250 grams, is less than 5%.

Over the period 2000 – 2015, the number of Dutch mushroom farms has decreased. A graph describing this development is given below. This graph is also shown on the supplement to this exam.



- 1p **c** Explain how it follows from this graph that the number of Dutch mushroom farms decreased exponentially.
- 4p **d** Calculate the half-life time of the number of mushroom farms during the period 2000 - 2015. Round your answer to the nearest month.

In the Dutch province of Noord-Brabant the number of mushroom farms has also decreased over the years. On the other hand, the average surface area per farm has increased. The relationship between the number of farms and the average surface area per farm is inversely proportional. On January 1st, 2017 there were 50 farms; the average surface area was 5400 m² per farm. On January 1st, 2020 there were 4 fewer of those farms.

- 2p **e** Calculate the average surface area per farm in m² on January 1st, 2020.

Question 5: Sound

The sound level is often expressed in terms of decibels (dB). This level can be calculated with the following formula:

$$S = 10 \cdot \log\left(\frac{I}{10^{-12}}\right)$$

Here:

- S is the sound level (in dB)
- I is the intensity of the sound (in Watt per m^2)

The intensity of the sound of a certain vacuum cleaner equals 10^{-5} Watt per m^2 .

- 1p **a** Show by using a calculation that the sound level of the vacuum cleaner is equal to 70 dB.

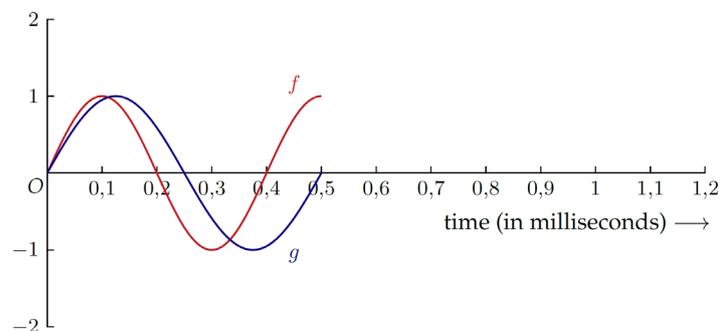
A well-known rule of thumb is the following: 'When the intensity of sound doubles, the sound level increases by approximately 3 dB'. This rule of thumb can be proved by rewriting the expression $10 \cdot \log\left(\frac{2 \cdot I}{10^{-12}}\right)$ into the form $10 \cdot \log\left(\frac{I}{10^{-12}}\right) + a$ where $a \approx 3$.

- 3p **b** Validate the rule of thumb in this manner and calculate a rounded to two decimals.

The formula for S can be rewritten into the form $I = 10^p \cdot 10^{-q}$.

- 4p **c** Calculate the values of p and q by rewriting the formula for S into the above form.

Sound propagates in the form of sound waves. Below you see the graphs of f and g . This figure is also shown on the supplement to this exam.



The graphs of f and g intersect on the horizontal axis at the origin.

- 2p **d** After how many milliseconds do the graphs intersect on the horizontal axis for the second time? You may use the supplement to this exam to obtain your answer.

Consider a third sound wave h such that:

- The amplitude of h is twice as large as the amplitude of f .
- The period of h is half of the period of f .
- At $t = 0$ the graph of h attains its maximum value.

Sound wave h can be described by a function of the form $h(t) = b \cdot \sin(c(t - d))$. Here t is the time in milliseconds.

- 4p **e** Give possible values for b , c and d . Explain clearly how you obtained your answer.

Question 6: Car manufacturing

Cars are usually manufactured in series. Because the production process becomes more efficient with each subsequent series, the number of men hours per car decreases with the serial number.



In this question we consider a type of car for which the following formula holds:

$$t = 1000 \cdot n^{-0.8}$$

Here t is the number of men hours necessary to manufacture a car of the n -th series.

- 2p **a** Calculate the percentage with which the number of men hours per car decreases between the first and the second series. Round your answer to the first decimal.
- 3p **b** Determine the derivative of t and explain using the formula of the derivative that the number of men hours decreases with each subsequent series.

The formula $t = 1000 \cdot n^{-0.8}$ can be rewritten into a form where n is expressed in terms of the number of men hours t .

- 3p **c** Express n in terms of t and simplify your formula.