BOSWELL-BÈTA

James Boswell Exam

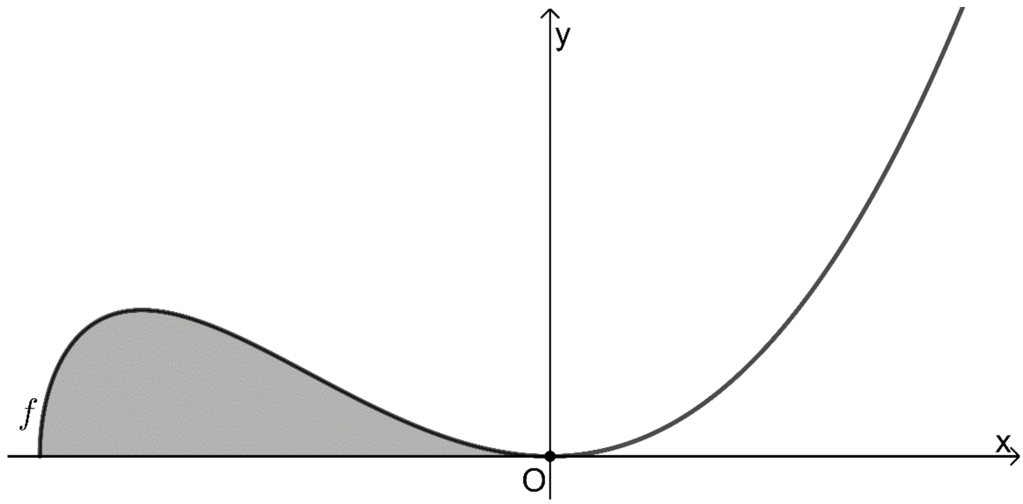
VWO Mathematics B – Practice exam 1

Date:	
Time:	3 hours
Number of questions:	6
Number of subquestions:	16
Number of supplements:	0
Total score:	81

- Write your name on every sheet of paper you hand in.
- Use a separate sheet of paper for each question.
- For each question, show how you obtained your answer either by means of a calculation or, if you used a graphing calculator, an explanation. Otherwise, no points will be awarded to your answer.
- Make sure that your handwriting is legible and write in ink. No correction fluid of any kind is permitted. Use a pencil only to draw graphs and geometric figures.
- You may use the following:
    - Graphing calculator (without CAS);
    - Protractor and compass;
    - Dictionary, subject to the approval of the invigilator.

Question 1. Given is the function $f(x) = x^2 \cdot \sqrt{x+1}$. In figure 1.1 the graph of f has been drawn.

Figure 1.1



Point $A\left(-\frac{3}{4}, \frac{9}{32}\right)$ is a point on the graph of f. Line ℓ is tangent to the graph of f at point A.

5p a. Show analytically that line ℓ is given by: $\ell: y = -\frac{3}{16}x + \frac{9}{64}$.

V is the area of the plane enclosed by the graph of f and the x-axis. In figure 1.1 area V has been shaded grey.

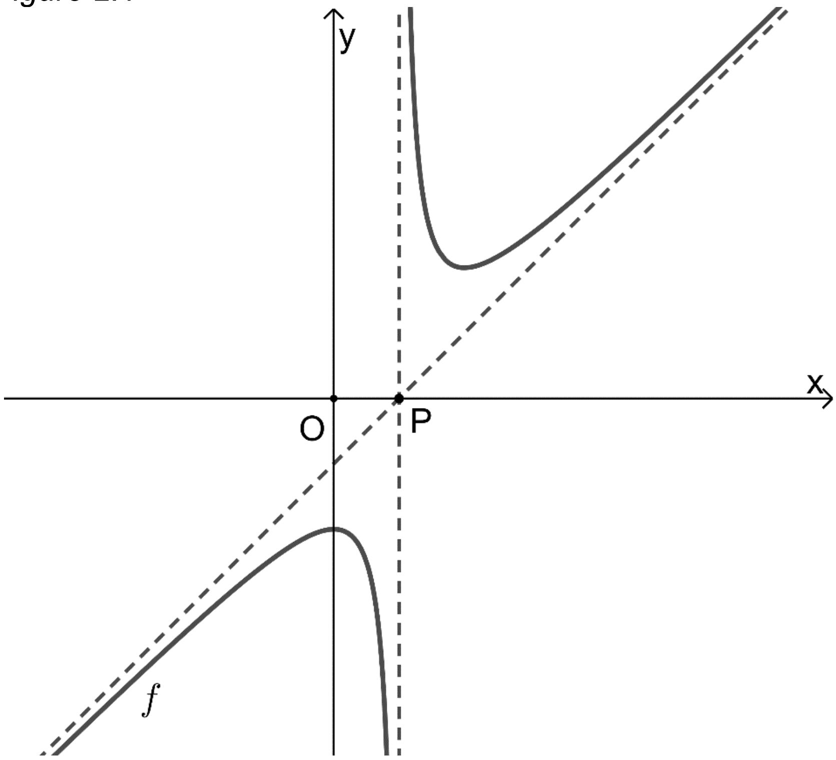
V is revolved around the x-axis.

6p b. Calculate analytically the volume of the corresponding solid of revolution.

Question 2. Given is the function $f(x) = \frac{x^2-2x+2}{x-1}$.

In figure 2.1 the graph of f has been drawn, together with its vertical and slant asymptote.

Figure 2.1



Point P has coordinates $(1, 0)$.

5p a. Prove that the asymptotes of the graph of f intersect each other at point $P(1, 0)$.

The graph of f is symmetric in point $P(1, 0)$ if for every value of a we have that:

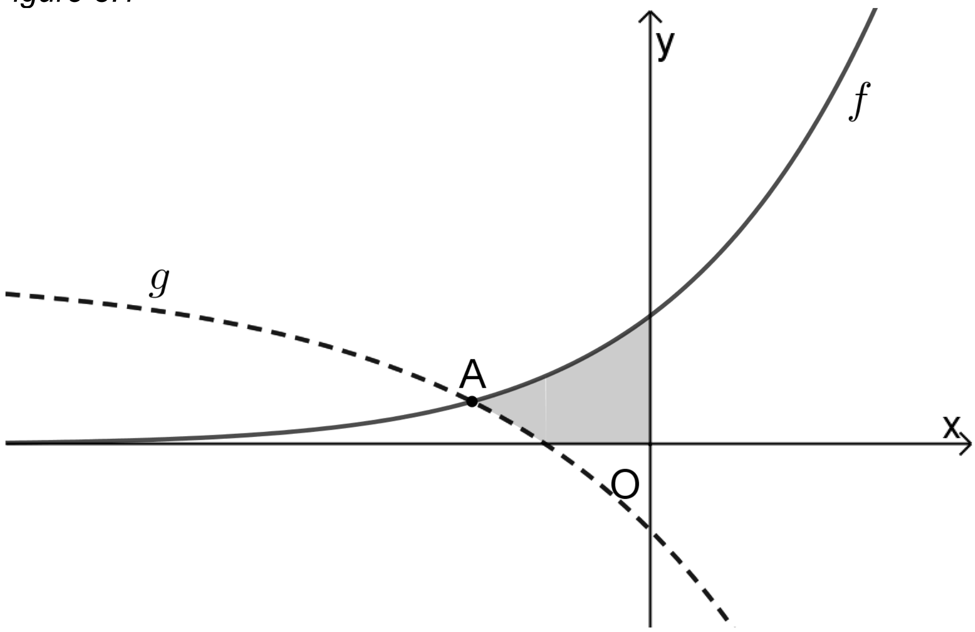
$$f(1 + a) = -f(1 - a)$$

5p b. Prove that the graph of f is symmetric in point $P(1, 0)$.

Question 3. Given are the functions:

$$f(x) = \frac{e^{2x}}{e^x + 1} \quad \text{and} \quad g(x) = \frac{2}{3} - e^x$$

Figure 3.1



The graphs of f and g intersect each other at only one point. We call this point A .

4p a. Show analytically that the x-coordinate x_A of point A is equal to $x_A = -\ln(2)$.

4p b. Prove that the function $F(x) = e^x - \ln(e^x + 1)$ is an antiderivative of $f(x)$.

V is the part of the plane enclosed by the graph of f, the graph of g, the x-axis and the y-axis.

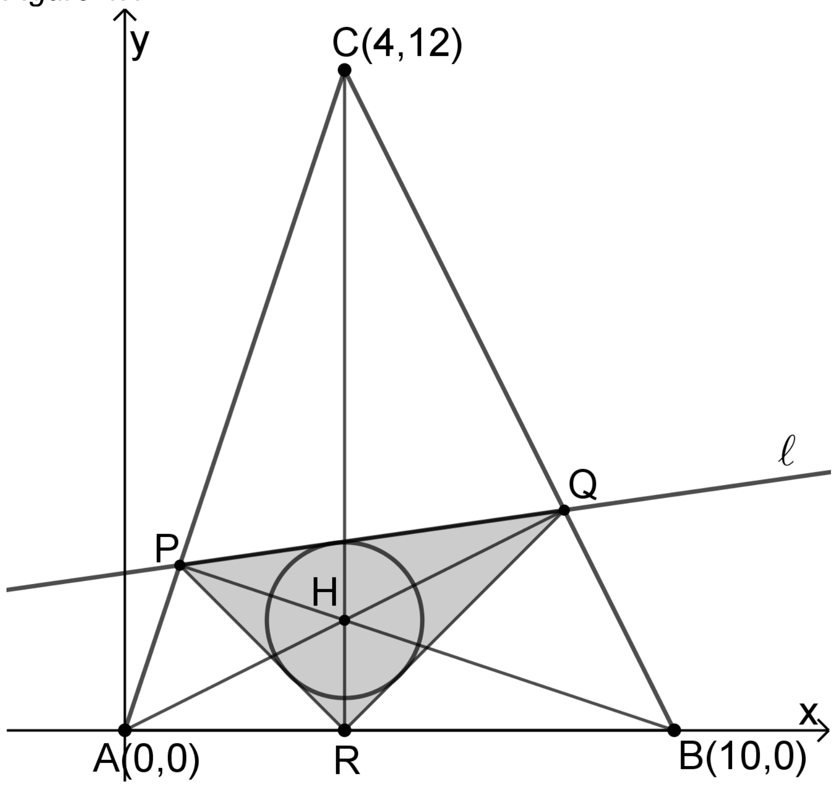
In figure 3.1 part V has been shaded grey.

6p c. Calculate algebraically the surface area of V. Give the analytical answer or round your answer to the third decimal.

Question 4. Given are line $\ell: 7y - x = 20$ and triangle ΔABC with $A(0,0)$, $B(10,0)$ and $C(4,12)$.

In figure 4.1 line ℓ and triangle ΔABC have been drawn.

Figure 4.1



Line segment AC intersects line ℓ at point P. Point P has coordinates $(1,3)$.

4p a. Prove that AP is perpendicular to BP.

Line segment BC intersects line ℓ at point Q.

4p b. Deduce that the coordinates of point Q are equal to $(8,4)$.

Line segments AQ and BP intersect each other at point H. See figure 4.1.

The line through points C and H intersects line segment AB at point R.

It turns out that:

- line segment CR is perpendicular to AB.
- point H is the centre of the inscribed circle of triangle ΔPQR.

(You do not have to prove this.)

6p c. Prove that the inscribed circle of triangle ΔPQR is given by:

$$x^2 - 8x + y^2 - 4y + 18 = 0$$

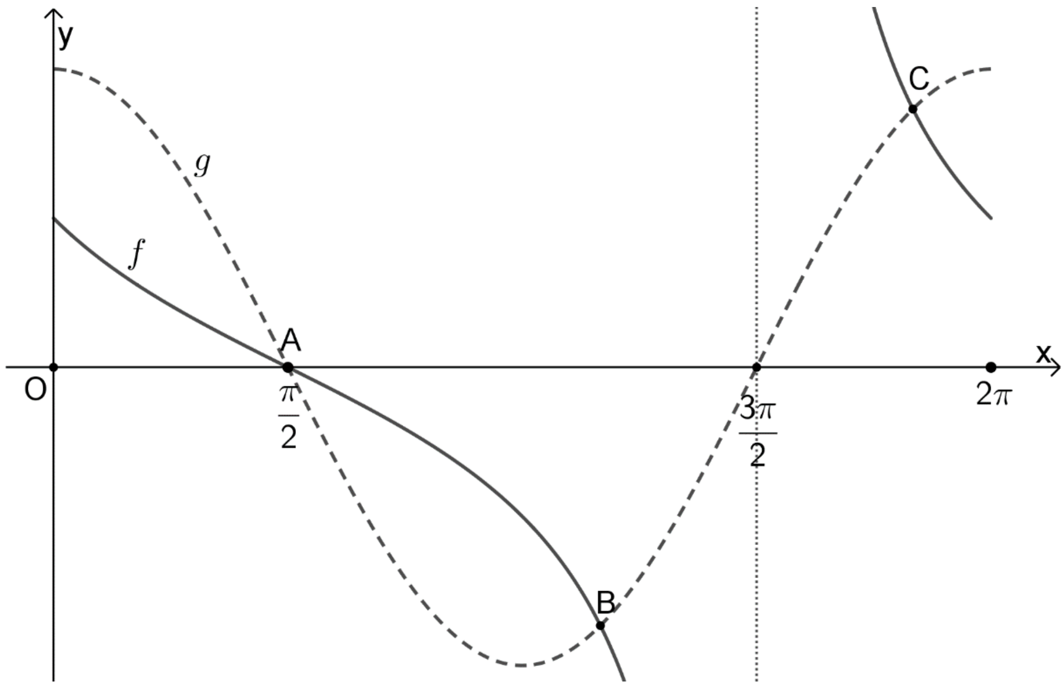
Question 5. On the interval $[0, 2\pi]$ the function f is given by:

$$f(x) = \frac{\cos(x)}{\sin(x) + 1} \qquad \left(x \neq \frac{3}{2}\pi\right)$$

The function g is given by: $g(x) = 2\cos(x)$.

In figure 5.1 the graphs of f and g have been drawn.

Figure 5.1



The graphs of the functions f and g intersect each other at points $A\left(\frac{1}{2}\pi, 0\right)$, B and C.

6p a. Calculate analytically the x-coordinates of points B and C.

5p b. Prove that:

$$f'(x) = \frac{-1}{\sin(x) + 1}$$

The line tangent to the graph of f at point $A\left(\frac{1}{2}\pi, 0\right)$ intersects the y-axis at point P.

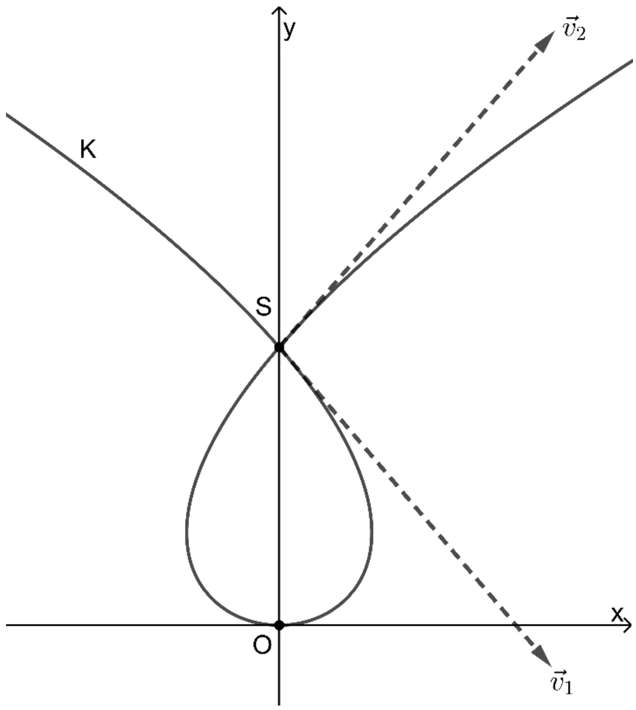
4p c. Show analytically that the distance between point A and point P is equal to $\frac{\pi}{4}\sqrt{5}$.

Question 6. The motion of point P through the plane is given by:

$$P: \begin{cases} x(t) = t^3 - 3t \\ y(t) = 2t^2 \end{cases}$$

The path of P is called curve K. In figure 6.1 curve K has been drawn.

Figure 6.1



Point P passes point $S(0,6)$ twice. The first time with velocity vector $\vec{v}_1$, the second time with velocity vector $\vec{v}_2$.

6p a. Calculate algebraically the angle in degrees between $\vec{v}_1$ and $\vec{v}_2$. Round your answer to the second decimal.

For three values of t the velocity vector $\vec{v}(t)$ of point P is perpendicular to the acceleration vector $\vec{a}(t)$ of point P.

7p b. Calculate analytically the coordinates of point P at these three values of t.

Curve K has two vertical tangent lines.

4p c. Calculate analytically the distance between these two tangent lines.

END OF EXAM