BOSWELL-BÈTA

James Boswell Exam

VWO Mathematics B – Practice exam 2

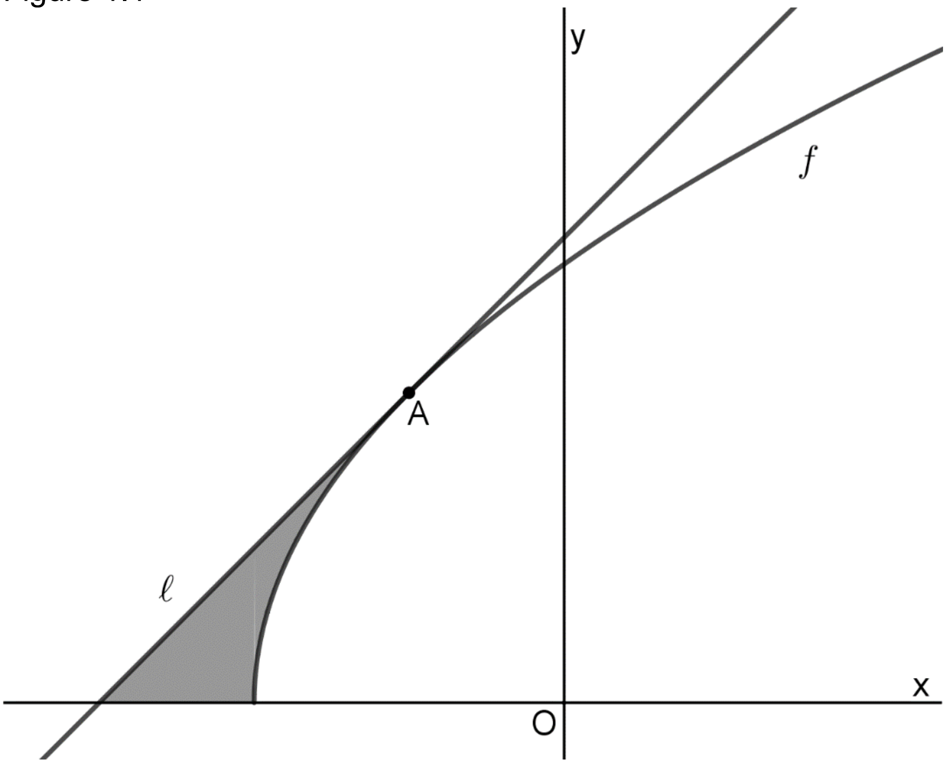
Date:	
Time:	3 hours
Number of questions:	6
Number of subquestions:	15
Number of supplements:	0
Total score:	82

- Write your name on every sheet of paper you hand in.
- Use a separate sheet of paper for each question.
- For each question, show how you obtained your answer either by means of a calculation or, if you used a graphing calculator, an explanation. Otherwise, no points will be awarded to your answer.
- Make sure that your handwriting is legible and write in black or blue ink. No correction fluid of any kind is permitted. Use a pencil only to draw graphs and geometric figures.
- You may use the following:
    - Graphing calculator (without CAS);
    - Protractor and compass;
    - Dictionary, subject to the approval of the invigilator.

Question 1. Let the function $f(x) = \sqrt{4x + 8}$ be given.

In figure 1.1 the graph of f has been drawn.

Figure 1.1



Line ℓ is tangent to the graph of f at point $A(-1, 2)$.

5p a. Prove that line ℓ is given by: $y - x - 3 = 0$.

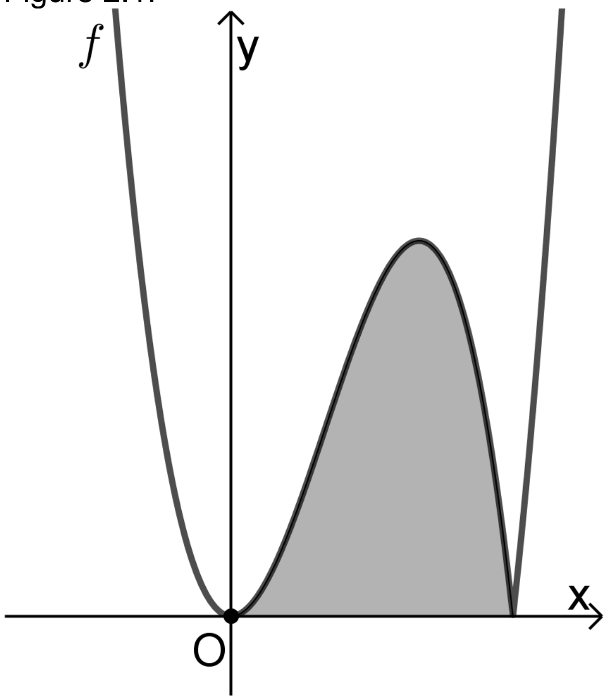
V is the part of the plane enclosed by the graph of f, line ℓ and the x-axis. In figure 1.1 area V has been shaded grey.

6p b. Calculate analytically the surface area of V.

Question 2. Let the function $f(x) = |x^3 - 3x^2|$ be given.

The graph of f has been drawn in figure 2.1.

Figure 2.1.



The graph of f and the line $y = 2x$ have a number of points in common.

6p a. Calculate analytically the number of points they have in common.

Area V is the part of the plane that is enclosed by the graph of f and the x-axis. In figure 2.1 area V has been shaded grey.

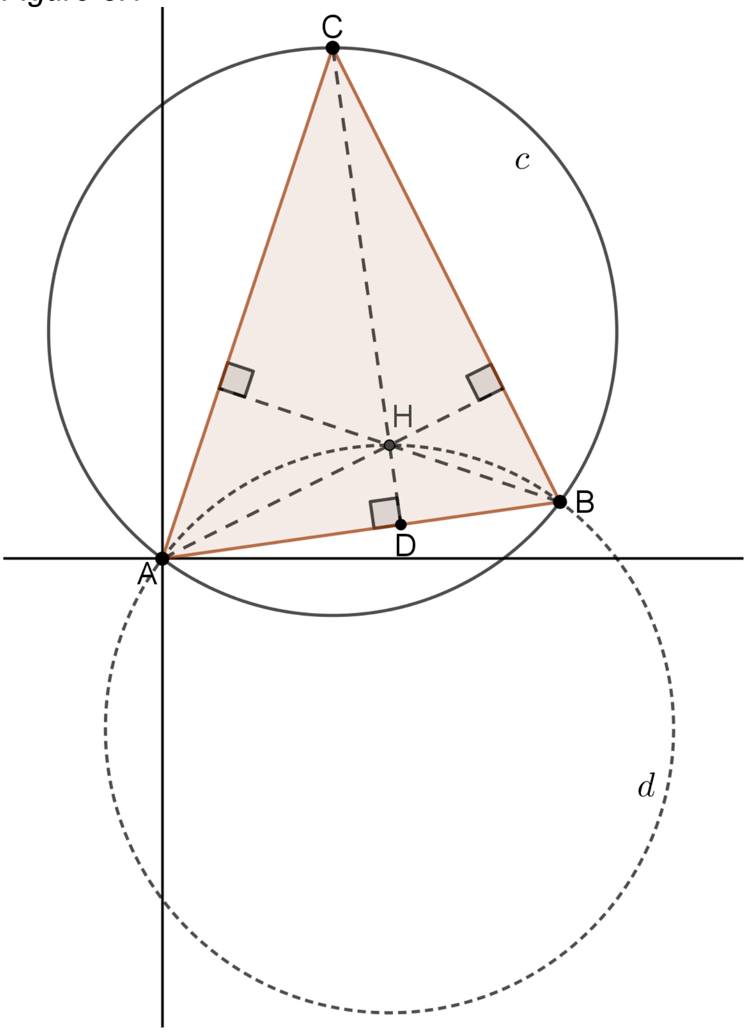
Suppose area V is revolved around the x-axis.

6p b. Calculate algebraically the corresponding volume obtained by this revolution.

Question 3. Given is circle $c: x^2 - 12x + y^2 - 16y = 0$. On circle c lie points $A(0,0)$, $B(14,2)$ and $C(6,18)$.

Circle c and triangle ΔABC have been drawn in figure 3.1.

Figure 3.1



The three altitude lines of ΔABC have been drawn as well. (An alttude line of a triangle goes from a vertex of the triangle to the opposite side of the triangle under an angle of $90°$.)

The altitude lines of triangle ΔABC intersect each other at point $H(8,4)$.

5p a. Prove that $\angle AHB = 135°$.

The line through points C and H intersects line segment AB at point D.

5p b. Calculate analytically the coordinates of point D.

Circle d is the circle going through points A, H and B. See again figure 3.1.

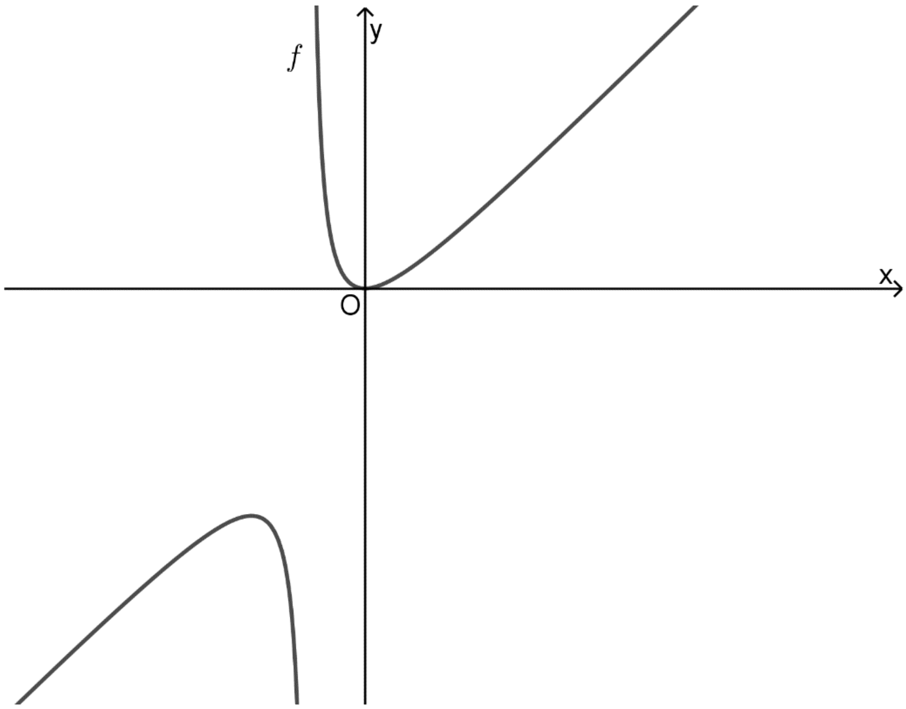
6p c. Prove that circle c has the same radius as circle d.

Question 4. The function f is given by:

$$f(x) = \frac{x^2}{x+1}$$

The graph of f has been drawn in figure 4.1.

Figure 4.1



4p a. Prove that:

$$f'(x) = \frac{x^2 + 2x}{x^2 + 2x + 1}$$

For certain values of p the line $l_p\colon y = -3x + p$ is tangent to the graph of f.

5p b. Calculate analytically these values of p.

The function g is given by:

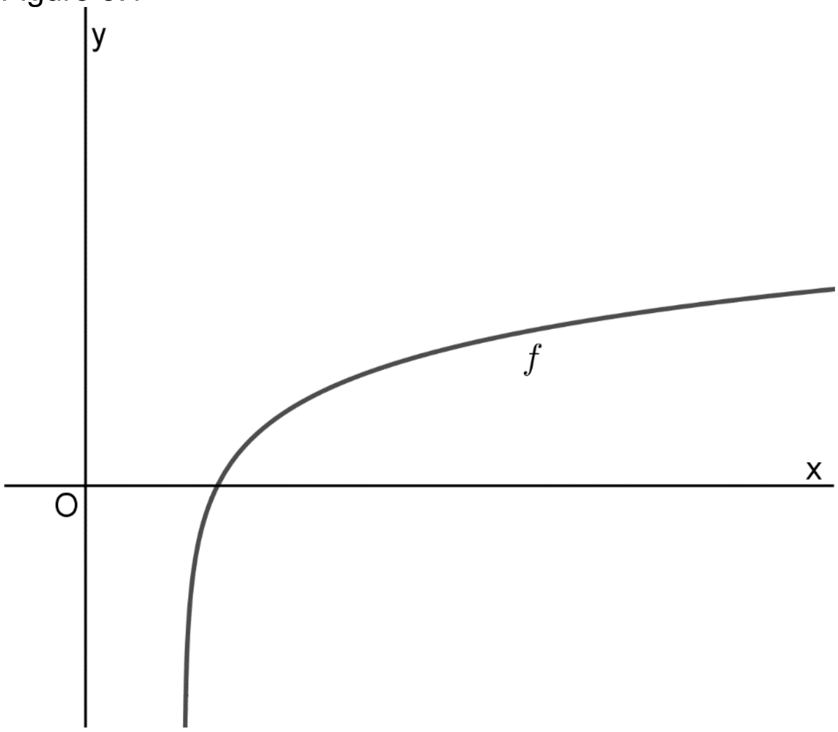
$$g(x) = \frac{x^3 - x^2}{x^2 - 1}$$

The graph of g is equal to the graph of f after point $P\left(1, \frac{1}{2}\right)$ has been removed from the graph of f.

5p c. Show analytically that this is indeed the case.

Question 5. In figure 5.1 the graph of a certain function f has been drawn.

Figure 5.1



The graph of f is obtained from the graph of $y = e^x$ by subsequently:

(I) Translating the graph 5 units upward.
(II) Applying a multiplication of a factor $\frac{1}{3}$ with respect to the x-axis.
(III) Mirroring the graph in the line $y = x$.

5p a. Show that the graph of f can be described by the formula: $f(x) = \ln(3x - 5)$.

The line ℓ: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 6 \end{pmatrix}$ is tangent to the graph of f.

6p b. Prove this.

Question 6. The motion of point P through the plane is given by the following equations:

$$\begin{cases} x(t) = \cos(t) \\ y(t) = \sin(2t) + \cos(t) \end{cases} \qquad (0 \leq t \leq 2\pi)$$

The trajectory of point P is called curve K. In figure 6.1 curve K has been drawn.

Figure 6.1

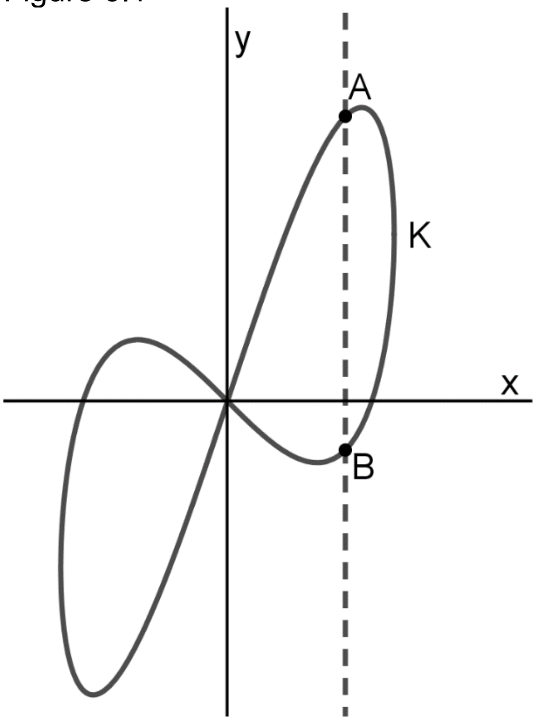
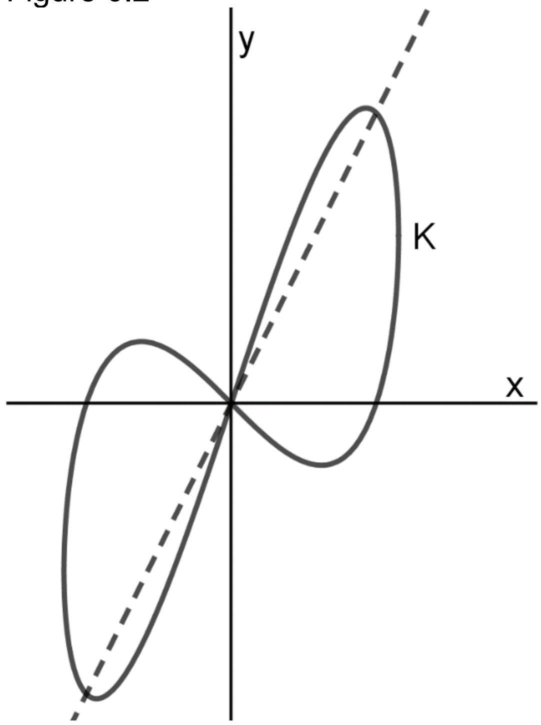


Figure 6.2



Curve K intersects the line $x = \frac{1}{2}\sqrt{2}$ at points A and B.

5p a. Calculate analytically the length of line segment AB.

Point P passes the origin $O(0,0)$ twice: the first time with velocity vector $\vec{v}_1$ and the second time with velocity vector $\vec{v}_2$.

6p b. Calculate algebraically the angle between vectors $\vec{v}_1$ and $\vec{v}_2$.

There are 4 values of t on the interval $[0, 2\pi]$ for which point P passes the line $y = 2x$. See figure 6.2.

7p c. Calculate analytically for which values of t point P is *above* the line $y = 2x$.

END OF EXAM