BOSWELL-BÈTA

James Boswell Exam

VWO Mathematics C – Practice exam 1

SOLUTION KEY

Date	
Time:	3 hours
Number of questions:	6
Number of subquestions:	21
Maximum score:	62

Subject-specific marking rules and guidelines

1. For each error or mistake in calculation a single point will be subtracted from the maximum score that can be obtained for that particular part of the question.
2. If a required explanation, deduction or calculation has been omitted or has been stated incorrectly 0 points will be awarded, unless otherwise stated in the solution key. This is also the case for answers obtained by the use of a graphic calculator. Answers obtained by the graphic calculator should indicate how the graphic calculator has been used to obtain the answer. Candidates must make sure they mention formulas applied or provide lists and calculation methods used in their answers.
3. If a notational error has been made, but the error can be seen to have no influence on the final result, no points will be deducted from the total score. If, however, it is not possible to determine that there is no influence on the final result a point will be deducted from the final score.
4. A particular mistake in the answer to a particular exam question will lead to a deduction of points only once, unless the question is substantially simplified by the mistake and/or when the solution key specifies otherwise.
5. A repeated mistake made in the answer to different exam questions will lead to a deduction of points each time such a mistake has been made, unless the solution key specifies otherwise.
6. If only one example, reason, explication, explanation or any other type of answer is required and more than one has been given, only the first answer given will be graded. If more than one example, reason, explication, explanation or any other type of answer is required, only the first answers are graded, up to and including the number of answers specified by the exam question.
7. If the candidate fails to give a required unit in the answer to a question a single point will be subtracted from the total score, unless the unit has been specified in the exam question.
8. If during intermediate steps results are rounded, resulting in an answer different from one in which non-rounded intermediate results are used, one point will be subtracted from the total score. Rounded intermediate results may, however, be noted down
Exceptions to this rule are those cases in which the context of the question requires the rounding of intermediate results. The <u>maximum</u> number of points deducted from the total score due to rounding errors is 2 for the entire exam.

Examples for the exceptions to rule 8.
Rounding off intermediate results can be forced by the context if, for example
- The amount of money for a single good has to be rounded to two decimals;
- The number of persons, things, etc. in a concrete situation (i.e. not for an average or expected value) has to be rounded to the nearest integer.

A required level of accuracy can be forced by the context if, for example
- The answer would not be distinguishable from a trivial answer. This can occur with the rounding of growth factors or probabilities to 0 or 1. A probability of $\left(\frac{1}{6}\right)^5$ may be rounded to 0.0001 but not to 0.000.

The forced rounding up or down of answers can occur, for example
- If the exam question specifies a minimum or maximum amount. (For example, if the question is: ‘What is the minimal distance an athlete has to jump to gain a certain number of points in a contest?’)

The above examples by no means exhaust all possible cases.

Question 1: American roulette

a	$P(\text{greater than } 9) = \frac{27}{38}$	2
b	$X =$ number of times you win when you play the first variant 20 times. $X \sim Bin\left(20, \frac{1}{38}\right)$ $P(X = 1) = binompdf\left(20, \frac{1}{38}, 1\right) = 0.3171$ *1pt for the correct probability of success, 1pt for binompdf and 1pt for the answer*	3
c	$Y =$ number of times you win when you play the second variant 20 times. $Y \sim Bin\left(20, \frac{18}{38}\right)$ $P(Y \leq 10) = binomcdf\left(20, \frac{18}{38}, 10\right) = 0.6777$ *1pt for the correct probability of success, 1pt for binomcdf and 1pt for the answer*	3
d	$Z =$ number of times you win when you play the second variant *n* times. $Z \sim Bin\left(n, \frac{18}{38}\right)$ $P(Z > 15) = 1 - P(Z \leq 15)$ $= 1 - binomcdf\left(X, \frac{18}{38}, 15\right)$ Investigate when this probability is at least 0.80 for the first time. If $n = 38$, then $P(Z > 15) = 0.7912$ If $n = 39$, then $P(Z > 15) = 0.8297$ So at least 39 times.	 1 1 1 1 1

Question 2: Air pressure

a	$g_{100 \text{ meters}} = \left(\frac{0.9670}{1.0133}\right)^{\frac{1}{4}} \approx 0.9884$ The formula is $p = 1.0133 \cdot 0.9884^h$ (where h is expressed in hundreds of meters)	2 1
b	1250 meters corresponds to $h = 12.5$ So $p = 1.0133 \cdot 0.9884^{12.5} \approx 0.8758$ bar. *When using the alternative:* $p = 1.0148 \cdot 0.9902^{12.5} \approx 0.8973$ bar.	1 1
c	$g_{500 \text{ meters}} = 0.9884^5 \approx 0.9433$ So the air pressure drops every 500 meters with (approximately) 5.7%. *When using the alternative:* $g_{500 \text{ meter}} = 0.9902^5 \approx 0.9520$. So the air pressure drops every 500 meters with (approximately) 4.8%.	2 1
d	The equation $1.0133 \cdot 0.9884^h = 0.6$ has to be solved. Describing how this equation can be solved. (algebraically or by using the GC) The answer: $h \approx 44.91$. So the height of the Matterhorn is 4491 meters. *When using the alternative:* $h \approx 53.36$. So the height is 5336 meters.	1 1 2

Question 3: Regular polygons

a	The polygon has 10 sides. $B_{10} = \frac{10-2}{10} \cdot 180 = 144°$	1 1
b	Show, with a calculation, that the amount by which the interior angle increases when n increases by 1 is not constant. For example: $B_3 = 60°$, $B_4 = 90°$, $B_5 = \frac{3}{5} \cdot 180° = 108°$	2
c	Investigate when $\frac{n-2}{n} \cdot 180$ is larger than 155 for the first time. When $n = 14$, the interior angles are equal to 154.3° When $n = 15$, the interior angles are equal to 156° So at least 15 sides.	 1 1 1
d	The limit is 180°. An explanation by using the GC or by reasoning along the lines of: 'If n becomes very large, then $B_n \approx \frac{n}{n} \cdot 180 = 180°$.'	2

Question 4: Increased temperatures

a	$X =$ average temperature in June, $X \sim Norm(15.3, 1.28)$ $P(X > 15.7) = normalcdf(15.7, 10^{99}, 15.35, 1.28) \approx 0.3923$ In 39.2% of the months of June the average temperature is higher than 15.7 °C	 2 1
b	$invNorm(0.25, 15.35, 1.28) \approx 14.49$. So 14.49 °C.	2
c	$Y =$ average temperature in November. $Y \sim Norm(5.94, \sigma)$ It has to be true that: $P(Y > 8{,}75) = 0{,}05$ z-score $= InvNorm(0.95) = 1.6449$ $\sigma = \frac{y-\mu}{z} = \frac{8.75-5.94}{1.6449} \approx 1.71$ °C.	 1 2

Question 5: Expressions

a	$2y + 10 = \frac{6 - x}{3}$ $2y + 10 = 2 - \frac{1}{3}x$ $2y = -8 - \frac{1}{3}x$ $y = -4 - \frac{1}{6}x$	1 1 1 1
b	$y^2 = \frac{1}{2}x - 9$ $\frac{1}{2}x = y^2 + 9$ $x = 2y^2 + 18$	1 1 1
c	$\frac{\sqrt{8} \cdot \frac{1}{2}}{\sqrt[3]{4}} = \frac{\sqrt{2^3} \cdot \frac{1}{2^1}}{\sqrt[3]{2^2}}$ $= \frac{(2^3)^{\frac{1}{2}} \cdot 2^{-1}}{(2^2)^{\frac{1}{3}}}$ $= \frac{2^{\frac{3}{2}} \cdot 2^{-1}}{2^{\frac{2}{3}}}$ $= \frac{2^{\frac{1}{2}}}{2^{\frac{2}{3}}}$ $= 2^{\frac{1}{2} - \frac{2}{3}} = 2^{-\frac{1}{6}}$	2 1 1 1

Question 6: Stressed out students

a	The percentage of students for which D does not hold is: $100 - 83 = 17\%$ So the minimum percentage is $60 - 17 = 43\%$	1 1
b	The percentage of students for which D **and** E do not hold is: $100 - 43 = 57\%$ So the minimum percentage is $70 - 57 = 13\%$	1 1
c	$D \Rightarrow E$: 'If a student experiences pressure or physical complaints, then the student is emotionally exhausted'. This proposition is **not true**, he or she could also experience other complaints. $\neg D \Rightarrow \neg E$: 'If a student does not experience pressure or physical complains, then the student is not emotionally exhausted'. This proposition is **true**, because if the student would be emotionally exhausted then he or she would experience pressure or physical complaints.	1 1 1 1