

James Boswell Exam VWO Mathematics C – Practice exam 2

SOLUTION KEY

Date

Time: 3 hours

Number of questions: 5

Number of subquestions: 19

Maximum score: 61

Subject-specific marking rules and guidelines

1. For each error or mistake in calculation a single point will be subtracted from the maximum score that can be obtained for that particular part of the question.
2. If a required explanation, deduction or calculation has been omitted or has been stated incorrectly 0 points will be awarded, unless otherwise stated in the solution key. This is also the case for answers obtained by the use of a graphic calculator. Answers obtained by the graphic calculator should indicate how the graphic calculator has been used to obtain the answer. Candidates must make sure they mention formulas applied or provide lists and calculation methods used in their answers.
3. If a notational error has been made, but the error can be seen to have no influence on the final result, no points will be deducted from the total score. If, however, it is not possible to determine that there is no influence on the final result a point will be deducted from the final score.
4. A particular mistake in the answer to a particular exam question will lead to a deduction of points only once, unless the question is substantially simplified by the mistake and/or when the solution key specifies otherwise.
5. A repeated mistake made in the answer to different exam questions will lead to a deduction of points each time such a mistake has been made, unless the solution key specifies otherwise.
6. If only one example, reason, explication, explanation or any other type of answer is required and more than one has been given, only the first answer given will be graded. If more than one example, reason, explication, explanation or any other type of answer is required, only the first answers are graded, up to and including the number of answers specified by the exam question.
7. If the candidate fails to give a required unit in the answer to a question a single point will be subtracted from the total score, unless the unit has been specified in the exam question.
8. If during intermediate steps results are rounded, resulting in an answer different from one in which non-rounded intermediate results are used, one point will be subtracted from the total score. Rounded intermediate results may, however, be noted down

Exceptions to this rule are those cases in which the context of the question requires the rounding of intermediate results. The maximum number of points deducted from the total score due to rounding errors is 2 for the entire exam.

Examples for the exceptions to rule 8.

Rounding off intermediate results can be forced by the context if, for example

- The amount of money for a single good has to be rounded to two decimals;
- The number of persons, things, etc. in a concrete situation (i.e. not for an average or expected value) has to be rounded to the nearest integer.

A required level of accuracy can be forced by the context if, for example

- The answer would not be distinguishable from a trivial answer. This can occur with the rounding of growth factors or probabilities to 0 or 1. A probability of $\left(\frac{1}{6}\right)^5$ may be rounded to 0.0001 but not to 0.000.

The forced rounding up or down of answers can occur, for example

- If the exam question specifies a minimum or maximum amount. (For example, if the question is: 'What is the minimal distance an athlete has to jump to gain a certain number of points in a contest?')

The above examples by no means exhaust all possible cases.

Question 1: Tip

a	To estimate the amount, use the class midpoints (2.5, 7.5, ...) $2 \cdot 2.5 + 17 \cdot 7.5 + 48 \cdot 12.5 + 29 \cdot 17.5 + 4 \cdot 22.5 = 1330$ euro.	1 2
b	$X =$ number of customers that gives a tip. $X \sim Bin(10, 0.8)$ $P(X \leq 8) = binomcdf(8, 10, 0.8) \approx 0.6242$ 1 pt for the description of the calculation, 1 pt for the answer.	2
c	For 5 dollars the tip equals: $F = 0.127 \cdot 5 + 1.21 = 1.845$ euro. So the tip is $\frac{1.845}{5} \cdot 100 = 36.9\%$ of the total bill. For 90 dollars the tip equals: $F = 0.127 \cdot 90 + 1.21 = 12.64$ euro. So the tip is: $\frac{12.64}{90} \cdot 100 \approx 14.04\%$ of the total bill. (So the percentage is higher for 5 euros)	1 1 1 1
d	The starting value of 1.21 has to be paid four times for 2. and for only once for 1. The variable part of $0,127R$ is the same in both cases. (So the total amount of tips is the highest for 2.)	2 1 1

Question 2: Soap

a	$X =$ weight of a piece of soap. $X \sim Norm(93, 1.4)$ $P(X < 90) = normalcdf(-10^{99}, 90, 93, 1.4) \approx 0.0161$. ($0.0161 < 0.02$), so Sanove complies with the European norm.	2 1
b	$T =$ weight of five pieces of soap, $T \sim Norm(\mu_T, \sigma_T)$ $\mu_T = 5 \cdot 93 = 465$ grams. $\sigma_T = \sqrt{5} \cdot 1,4 (\approx 3,13)$ grams. $P(T > 470) = normalcdf(470, 10^{99}, 465, \sqrt{5} \cdot 1,4) \approx 0.0551$.	1 1 2
c	$P(\text{weight below } \mu) = \frac{1}{2}$. $P(\text{all 10 below } \mu) = \left(\frac{1}{2}\right)^{10} \approx 0.0010$	1 2
d	$P(\text{recalibration}) = 1 - P(\text{all pieces of soap above } \mu - 3\sigma)$ $\mu - 3\sigma = 88.8$ grams. $P(\text{one piece of soap above } \mu - 3\sigma) = normalcdf(88.8, 10^{99}, 93, 1.4) \approx 0.9987$. $P(\text{all pieces above } \mu - 3\sigma) = 0.9987^{10} (\approx 0.9866)$. So we have $P(\text{recalibration}) = 1 - 0.9987^{10} \approx 0.0134$. <i>Alternative solution</i> $Y =$ pieces of soap that weigh less than $\mu - 3\sigma$. $Y \sim Bin(10, p)$ $\mu - 3\sigma = 88.8$ grams. The probability of success is $p = normalcdf(-10^{99}, 88.8, 93, 1.4) \approx 0.0013$ $P(Y \geq 1) = 1 - P(Y = 0)$ $= 1 - binompdf(10, 0.0013, 0) \approx 0.0134$	1 1 1 1 1 1 1 2

Question 3: Expressions

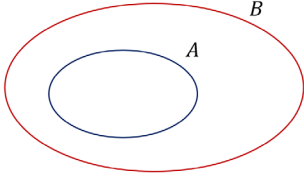
a	$-4 - y = \frac{1}{2}x - 1\frac{1}{2}$	1
	$-y = \frac{1}{2}x + 2\frac{1}{2}$	1
	$y = -\frac{1}{2}x - 2\frac{1}{2}$	1
b	$y = \sqrt[3]{x^5} \cdot 25 \cdot x^8$	2
	$y = x^{\frac{5}{3}} \cdot 25 \cdot x^8$	1
	$y = 25 \cdot x^{\frac{9^2}{3}}$	1
c	$3x + 12 = 2^y$	2
	$3x = 2^y - 12$	1
	$x = \frac{1}{3} \cdot 2^y - 4$ (of een equivalente uitdrukking)	1

Question 4: Flu epidemic¹

a	The period week 50 – week 14 is 16 weeks long.	1
	Every week the number of patients increases with $\frac{170-25}{16} = 9.0625$.	1
	January 1 th 2018 is 3 weeks after the beginning of week 50.	1
	So $25 + 3 \cdot 9.09625 \approx 52$ persons per 100 000 inhabitants.	1
b	$g_{week} = \left(\frac{170}{25}\right)^{\frac{1}{16}} \approx 1.127$.	2
	So $N = 25 \cdot 1.127^t$ (with t in weeks and $t = 0$ at the start of week 50 in 2017)	1
c	$N = 25 \cdot 1.127^3 \approx 36$ persons per 100 000 inhabitants. <i>When using the alternative formula: $N = 21 \cdot 1.138^3 \approx 31$ persons per 100 000 inhabitants.</i>	2
d	The equation $25 \cdot 1.127^t = 51$ has to be solved.	1
	Solving this equation (algebraically or by using the GC) yields $t \approx 5.96$.	1
	So week 3 of 2018. <i>When using the alternative formula: $t \approx 6.86$. So week 4 of 2018</i>	1
e	$g_{week} = 1.213$	1
	$g_{day} = (1.213)^{\frac{1}{7}} \approx 1.028$	1
	So with 2.8% per day.	1

¹ If someone is counting 'incorrectly' at most 2 points will be subtracted, for instance if a candidate writes down that the period week 50 – week 14 is 17 weeks long.

Question 5: Cruijff's logic

<p>a</p>	<p>$\neg B \Rightarrow \neg A$. $\neg B \Rightarrow \neg A$ is a logical consequence of $A \Rightarrow B$. If someone does not show skill X, then he does not possess this skill, because from the first part it follows that if he would have possessed the skill, he would also have showed it.</p> <p><i>Alternative solution</i> $\neg B \Rightarrow \neg A$ Drawing a Venn-diagram that corresponds to $A \Rightarrow B$:</p>  <p>Someone who does not show skill X is outside of B in the Venn-diagram. Then the person is also outside of A, so he does not possess the skill. So $\neg B \Rightarrow \neg A$ is a logical consequence of $A \Rightarrow B$.</p>	<p>1 1 1 1 1</p>
<p>b</p>	<p>Someone does not have to show all skills (or can't show all skills) that he possesses. So if he doesn't show a certain skill, you can't be sure if he possesses the skill or not.</p>	<p>2</p>
<p>c</p>	<p>$Q \Rightarrow P$. The proposition $Q \Rightarrow P$ conveys no information about the situation where no one has scored ($\neg Q$). The conclusion is that you don't know if someone has shot.</p>	<p>1 1 1</p>