

James Boswell Exam

VWO Mathematics C – Practice exam 1

Date:

Time: 3 hours

Number of questions: 6

Number of subquestions: 21

Maximum score: 62

- Write your name on **every sheet of paper** you hand in.
- For each question, show how you obtained your answer either by means of a calculation or, if you used a graphing calculator, an explanation. Otherwise, no points will be awarded to your answer.
- Make sure that your handwriting is legible and write in ink. No correction fluid of any kind is permitted. Use a pencil only to draw graphs and geometric figures.
- You may use the following:
 - Graphing calculator (without CAS);
 - Drawing utensils;
 - List of formulas
 - Dictionary, subject to the approval of the invigilator.

Question 1: American roulette

Roulette is a popular casino game. In this game a little ball is rolled in a turning wheel. When the little ball comes to halt, it falls into one of the numbered pockets of the turning wheel.

In American roulette there are 38 numbered pockets. Two of these pockets are coloured green and are numbered 0 and 00. The other pockets are numbered 1 through 36. Half of these pockets has the colour red, half of these pockets has the colour black.



In this question we consider two variants of playing American roulette, which we call variant 1 and variant 2.

variant 1

You pay € 1 each time you play the game. Upon playing, you put your money on one of numbers 1 through 36. If the little ball falls into the pocket that corresponds to the number of your choice, you are paid out € 36. If this is not the case you lose your € 1.

- 2p **a** Calculate the probability the little ball falls into a pocket that has a number greater than 9.

Suppose you play **variant 1** of American roulette 20 times.

- 3p **b** Calculate the probability that out of these 20 times you win precisely once. Round your answer to four decimals.

Another way of playing American roulette is the following:

variant 2

You pay € 1 each time you play the game. Upon playing, you put your money on either the colour red or the colour black. If the little ball falls into a pocket that has the colour of your choice you are paid out € 2. If this is not the case, you lose your € 1.

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Suppose you play **variant 2** of American roulette 20 times.

- 3p **c** Calculate the probability that out of these 20 games you win at most 10 times. Round your answer to four decimals.
- 5p **d** Investigate how many times you have to play this game so that the probability that you win more than 15 times is at least 80%.

Question 2: Air pressure

The air pressure on earth depends on the height above sea level. On a high ski slope the air pressure is lower than at sea level.

In the table below the air pressure is given at different heights above sea level.

Height above sea level (in km)	0	0,1	0,2	0,3	0,4
Air pressure (in bar)	1,0133	1,0015	0,9898	0,9784	0,9670

There exists an exponential relationship between the air pressure and the height above sea level. This means we can express the pressure p using a formula of the form $p = p_0 \cdot g^h$ where p is the air pressure (in bar), p_0 is the air pressure at sea level (in bar) and h the height above sea level (in hundreds of meters).

- 3p **a** Determine this formula for p . Use the values at 0 km and 0,4 km to calculate g . Round g to four decimals.

If you could not find the answer to question a you may use the (incorrect) formula $p = 1,0148 \cdot 0,9902^h$ in the remainder of this question.

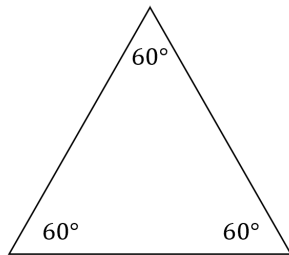
- 2p **b** Calculate the air pressure at 1250 meters above sea level. Round your answer to four decimals.
- 3p **c** Calculate with which percentage the air pressure decreases for each 500 meter increase in height. Round your answer to one decimal.

A mountain climber stands on top of the Matterhorn mountain in Switzerland. His weather glass (barometer) indicates that the air pressure is equal to 0,6 bar.

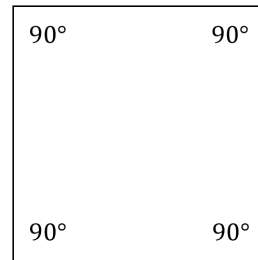
- 4p **d** Calculate the height of the Matterhorn. Give your answer rounded to the nearest meter.

Question 3: Regular polygons

A so-called regular polygon is any geometric figure for which all sides have equal length. Two examples are the equilateral triangle and the square. If the number of sides of the regular polygon increases, the interior angles of the polygon increase as well. For an equilateral triangle the internal angles are equal to 60° . For a square all internal angles are equal to 90°



An equilateral triangle.



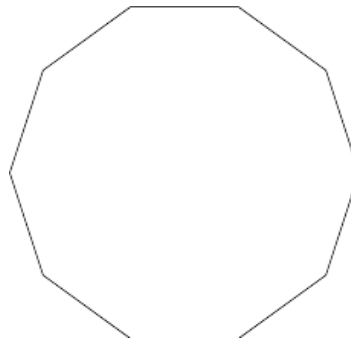
A square.

The relationship between the number of sides and the number of degrees of the internal angles of the polygon can be described by a numeric sequence. The direct formula of this sequence equals:

$$B_n = \frac{n-2}{n} \cdot 180 \text{ with } n \geq 3$$

Here B_n is the number of degrees of the internal angles of a regular polygon with n sides.

- 2p a Calculate the number of degrees of the internal angles of the following polygon:



- 2p b Show using a calculation that the sequence B_n is **not** an arithmetic sequence.
- 3p c What is the least number of sides a regular polygon has to have for the internal angles to be larger than 155° ?
- 2p d The number of degrees of the internal angles of the regular polygons approaches a certain limit as the number of sides n increases. Determine the value of this limit.

Question 4: Increased temperatures

In 1901 the Royal Dutch Institute of Meteorology KNMI (Koninklijk Nederlands Meteorologisch Instituut) started with the systematic registration of the temperature in the Netherlands. They determined that the average temperature during the month June is normally distributed with a mean of 15,35 °C and a standard deviation of 1,28 °C.

- 3p **a** Which percentage of the months June have an average temperature higher than 15,7 °C? Round your answer to one decimal.

June 1991 is one of the 25% coldest months of June that the KNMI has monitored.

- 2p **b** What is the highest possible value of the average temperature of this month? Round your answer to two decimals.

According to measurements performed by the KNMI, the average temperature of the month November is also normally distributed with a mean of 5,94 °C. According to a meteorologist only in 5% of the months November has the average temperature been higher than 8,75 °C.

- 3p **c** Calculate the standard deviation of the average temperature of the months November. Round your answer to two decimals.

Question 5: Expressions

- 4p **a** Write $\frac{6-x}{3} = 2(y + 5)$ in the form $y = ax + b$

- 3p **b** Write $y = -\sqrt{\frac{1}{2}x - 9}$ in the form $x = cy^2 + d$

- 5p **c** Write $\frac{\sqrt{8} \cdot \frac{1}{2}}{\sqrt[3]{4}}$ as a single power of 2.

Question 6: Stressed out students

On January 28th, 2018 the following news item was published on NOS.nl

Students who borrow money to attend university are more often extremely tired, emotionally exhausted and worried they will get a burn-out. They also refrain from social activities more often due to the costs involved with these activities, according to a student survey that has been performed for the Dutch Student Organization (ISO).

563 students have been surveyed. 83 percent of all the students who borrow money to attend university experience pressure or physical complaints. 60 percent is emotionally exhausted and 70 percent is worried about getting a burn-out.

According to the ISO, the survey shows that the necessity to borrow money has a large impact on the students. "Borrowing money not only causes financial problems, but also emotional ones. This survey shows this is to be taken as a matter of fact," says chairman Tom van den Brink.

In the second paragraph of the news article several percentages are given. These percentages involve three symptoms that students who need to borrow money to attend university may or may not have:

- D : the student experiences pressure or physical complaints.
- E : the student is emotionally exhausted.
- B : the student has worries of getting a burn-out.

The percentages mentioned in the news article can be used to determine the percentages of students having one or more of these symptoms. For the moment we assume that these symptoms do not rule each other out nor imply each other.

2p **a** Calculate the **minimum** percentage of students who borrow money and have both symptoms D and E .

2p **b** Calculate the **minimum** percentage of students who borrow money and have all three symptoms.

An expert claims that emotional exhaustion is an example of a physical complaint, so that symptom E implies symptom D meaning $E \Rightarrow D$. Assume this is indeed the case.

4p **c** Express the propositions $D \Rightarrow E$ and $\neg D \Rightarrow \neg E$ as sentences in English and explain for each of them if they are true or not.

End of exam