BOSWELL-BÈTA

James Boswell Exam

VWO Mathematics C – Practice exam 2

Date:	
Time:	3 hours
Number of questions:	5
Number of subquestions:	19
Number of supplements:	0
Maximum score:	61

- Write your name on **every sheet of paper** you hand in.
- For each question, show how you obtained your answer either by means of a calculation or, if you used a graphing calculator, an explanation. Otherwise, no points will be awarded to your answer.
- Make sure that your handwriting is legible and write in ink. No correction fluid of any kind is permitted. Use a pencil only to draw graphs and geometric figures.
- You may use the following:
    - Graphing calculator (without CAS);
    - Drawing utensils;
    - List of formulas;
    - Dictionary, subject to the approval of the invigilator.

Question 1: Tip

Sandra is a waitress at a local bar. During a hundred working days she kept a score of the height of tips she received during the day. The result is given in the following table:

Amount (in €)	$[0, 5\rangle$	$[5, 10\rangle$	$[10, 15\rangle$	$[15, 20\rangle$	$[20, 25\rangle$
Number of days	2	17	48	29	4

You can read from the table that Sandra received at least 5 euros but less than 10 euros on 17 out of 100 days. The table can be used to estimate the total amount of tips Sandra has received.

3p **a** Estimate this amount.

Of all the customers Sandra attended in the mentioned hundred days, 80% has given her a tip. Assume that this percentage also holds for the upcoming days.

2p **b** Calculate the probability that of the following 10 customers at most 8 will give Sandra a tip. Round your answer to the fourth decimal.

In the United States one is used to give tips in bars and restaurants. The psychologist L. Green investigated the relationship between the height of the bill and the height of the given tip. His research yielded the following formula:

$$F = 0.127R + 1.21$$

Here F is the height of the tip and R is the height of the bill (both in dollars).

Using Green's formula you can calculate the height of the tip as a percentage of the height of the bill.

4p **c** Show using a calculation that this percentage is larger for a bill of 5 dollars than for a bill of 90 dollars.

Four people ate at a certain restaurant. They consider two ways of paying the bill:

1. They each ask for a separate bill.
2. They divide the total amount of the bills by four and pay the same amount each.

3p **d** Argue in which of these two ways the total amount of tips is highest by using Green's formulas. An example with numbers is **not** sufficient.

Question 2: Soap

The firm Sanove produces pieces of soap. The pieces of soap are produced by machines. The machines are calibrated in such a way that pieces of soap are normally distributed with a mean of 93 grams and a standard deviation of 1.4 grams.

The European norm is that at most 2% of pieces of soap may weigh less than 90 grams.

3p **a** Perform a calculation to investigate whether the soap produced by Sanove complies with the European norm.

4p **b** Calculate the probability that five pieces of soap together weigh more than 470 grams. Round your answer to the fourth decimal.

The pieces of soap produced by the firm's machines may not produce too many pieces of soap with a weight that is too low. To check this, the quality department of the firm randomly selects ten pieces of soap each day. When each of the ten pieces of soap weighs less than 93 gram, the production machines have to be recalibrated.

3p **c** Calculate the probability that the machines need to be recalibrated. Round your answer to the fourth decimal.

The ten randomly chosen pieces of soap are also inspected in a second way: if there is at least one piece of soap with a weight that is more than 3 standard deviations below the mean of 93 grams, then the machine will be recalibrated.

5p **d** Calculate the probability that a correctly functioning machine will be recalibrated for this reason. Round your answer to the fourth decimal.

Question 3: Expressions

3p **a** Express $-4 - y = \frac{1}{2}(x - 3)$ in the form $y = ax + b$.

4p **b** Express $y = \sqrt[3]{x^5} \cdot (5x^4)^2$ in the form $y = ax^n$.

4p **c** Rewrite the expression $\log_2(3x + 12) = y$ in such a way that x is expressed in terms of y.

Question 4: Flu epidemic

Research bureau Nivel registers the number of people in The Netherlands having the flu at the beginning of each week. The number of people having the flu usually increases substantially during the winter season.

In the table on the right the number of people having the flu has been given at the beginning of week 50 in 2017 and at the beginning of week 14 in 2018.

	Number of people having the flu (in persons per 100 000 habitants)
week 50 – 2017	25
week 14 – 2018	170

First suppose the number of people having the flu increased **linearly**.

4p **a** Calculate the number of people per 100 000 inhabitants having the flu on 1 January 2018 using linear interpolation. Round your answer to the nearest whole number.

According to the research bureau the number of people having the flu has actually not increased linearly but **exponentially**. In the rest of this question we assume exponential growth for the number of people having the flu.

A formula of the form $N = b \cdot g^t$ can be constructed that fits the data given in the table. Here N is the number of persons per 100 000 inhabitants having the flu, and t is the time in weeks. $t = 0$ corresponds to the beginning of week 50 in 2017.

4p **b** Construct this formula. Round g to the third decimal.

*If you cannot answer question **b** you can use the (incorrect) formula $N = 21 \cdot 1.138^t$ in the remainder of this question.*

3p **c** Calculate how many people (in persons per 100 000 habitants) had the flu on 1 January 2018. Round your answer to nearest whole number.

When 51 or more people per 100 000 habitants have the flu there is a flu epidemic.

4p **d** Determine the first week in which an epidemic occurred according to the formula.

In 1998-1999 a serious flu epidemic occurred. At the beginning of 1999 the number of people having the flu increased with 21.3% every week. Assume that this growth is exponential.

3p **e** Calculate the percentage with which the number of people having the flu increased each day. Round your answer to the first decimal.

Question 5: Cruijff's logic

Former Dutch football (soccer) player and coach Johan Cruijff is known for his peculiar expressions of 'logic'. In an interview held in 2013 Cruijff said the following:

> "If I ask you: 'Show me what you can do', you will show what you can do. But then I immediately know what you cannot do, because you will not show that."

In this question we will analyze this logic further. To do so, we will restrict ourselves, for the time being, to a particular skill which we call X. We distinguish the following assumptions:

- A: someone possesses skill X.
- B: someone shows skill X.

The first part of Cruijff's statement can be understood as follows: 'If you possess skill X, you will show skill X'. In symbols: $A \Rightarrow B$. The second part of Cruijff's statement is: 'if someone does not show skill X, he does not possess skill X'.

3p **a** Express the second part of Cruijff's statement in terms of logical symbols and indicate whether Cruijff's second statement is a logical consequence of the first statement. Explain your answer.

2p **b** Explain why you can criticize Cruijff's statement.

Another well-known statement made by Cruijff is:

> "You have to shoot, otherwise you cannot score."

To analyze this statement, we introduce two abbreviations:

- P: someone scored.
- Q: someone has shot.

Cruijff's statement can then be reformulated as: 'If someone scored, then someone has shot'.

3p **c** Write the statement 'If someone scored, then someone has shot' using logical symbols and the abbreviations P and Q. Subsequently indicate what this statement says about a football (soccer) match during which no one scored.

End of exam